Real-Time Trajectory Generation and Tracking for Cooperative Control Systems

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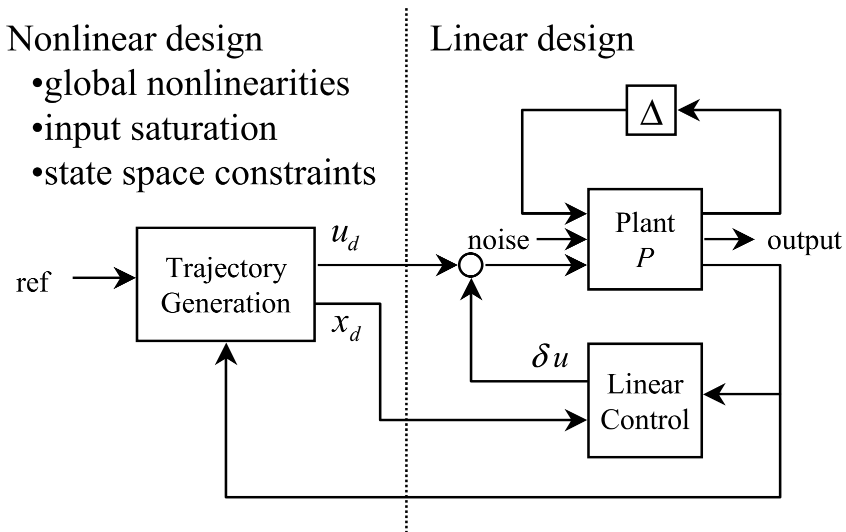
Outline

I. Review of previous work in trajectory generation and tracking

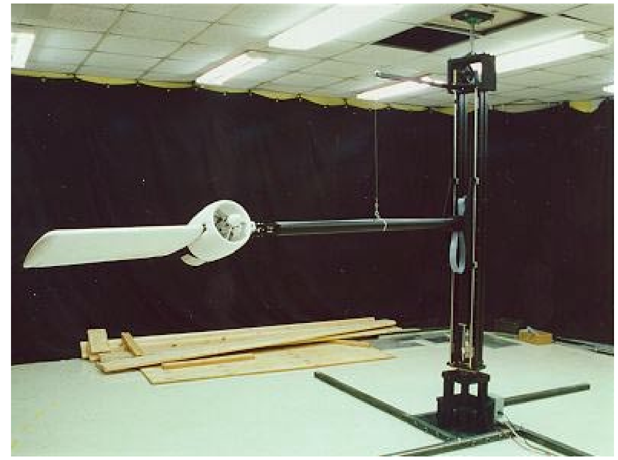
II. Cooperative trajectory generation via optimization-based control

III. Research plan and integration

Trajectory Generation and Tracking Using Differential Flatness



Caltech Ducted Fan



Approach: Two Degree of Freedom Design

- Use online trajectory generation to construct feasible trajectories
- Use (scheduled) linear control for performance and robustness
- For many flight vehicles, system is differentially flat $\Rightarrow$ reduce dynamic system to algebraic equivalent and generate feasible trajectories in real time

Results (PRET + SEC)

- Framework for exploiting differential flatness in real-time trajectory generation
- Necessary and sufficient conditions for flatness classes of (mechanical) systems
- NTG software package for finite time optimal control in presence of constraints
- Implementation and testing on Caltech ducted fan
- Transitions in progress to SEC, Raytheon

Real-Time Trajectory Generation / Optimization

$$\dot{x} = f(x,u)$$

Collocation

$$(x,u) = \sum \alpha_i \psi^i(t)$$
$$\dot{x}(t_i) = f(x(t_i),u(t_i))$$

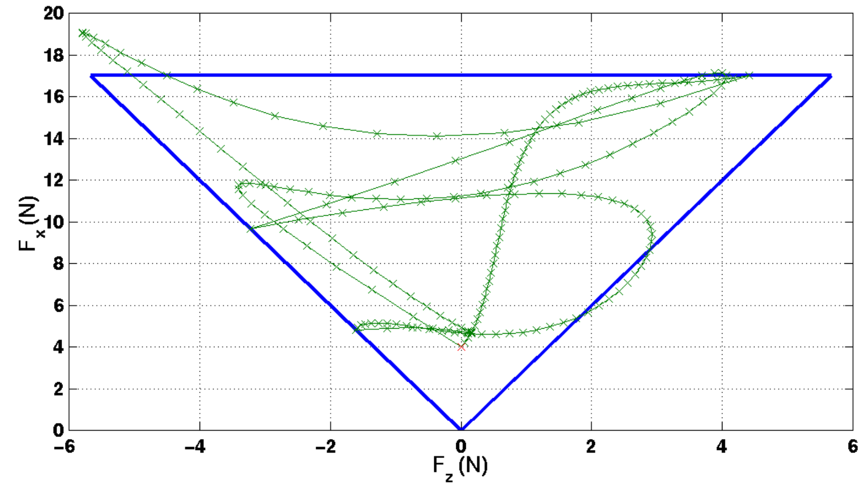
Flatness

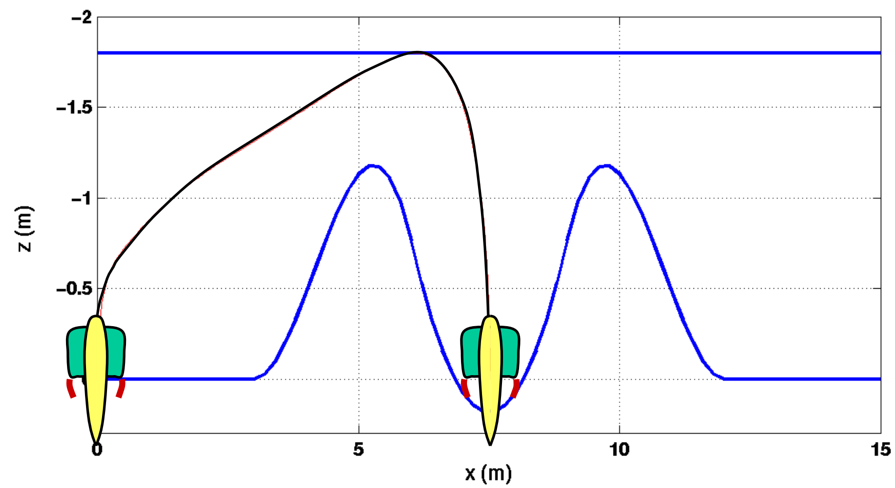
$$z = z(x,u,\dot{u},\ldots,u^{(p)})$$
$$x = x(z,\dot{z},\ldots,z^{(q)})$$
$$u = u(z,\dot{z},\ldots,z^{(q)})$$
$$z = \sum \alpha_i \psi^i(t)$$

Quasi-collocation

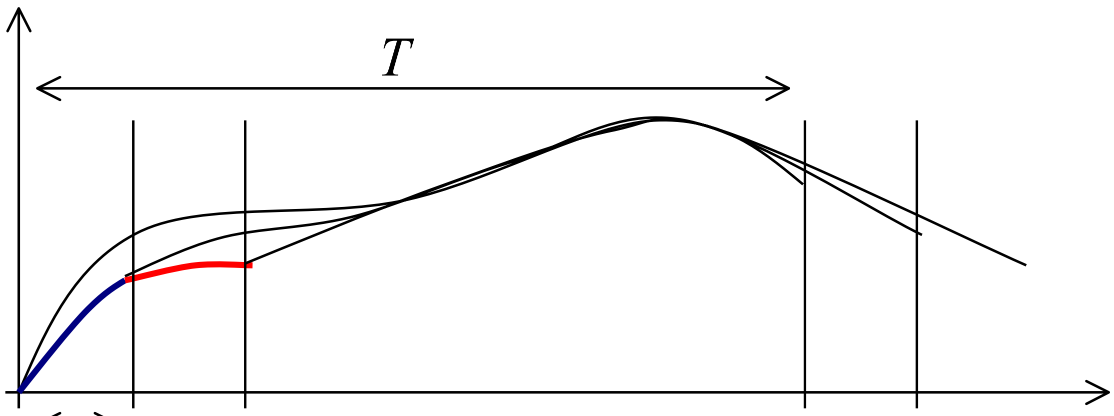
$$y = h(x)$$
$$(x,u) = \Gamma(y,\dot{y},\ldots,y^{(q)})$$
$$0 = \Phi(y,\dot{y},\ldots,y^{(p)})$$

Ducted Fan Terrain Avoidance





Optimization-Based Control: MPC + CLF



$$u_{[t,t+\Delta T]} = \arg\min \int_t^{t+T} q(x(\tau),u(\tau))\,d\tau + V(x(t+T))$$

$$x_0 = x(t) \quad x_f = x_d(t+T)$$

$$\dot{x} = f(x,u) \quad g(x,u) \le 0$$

Online control customization

- System: f(x,u)
- Constraints/environment: g(x,u)
- Misssion: L(x,u)

Update in real-time to achieve *reconfigurable* operation

Theory: MPC + CLF Approach

$$J_T^*(x_0) = \min_{u(\cdot)} \int_0^T q(x(\tau), u(\tau))\, d\tau + V(x(T))$$

Finite horizon

CLF
≈ "cost to go"

$$\inf_u \; (\dot{V} + q)(x, u) \;\leq\; 0$$

Basic Idea

- Use online models to compute receding horizon optimal control
- CLF-based terminal cost gives stability + short time horizons

Properties

- Can prove stability (in absence of constraints)
- Incremental improvement property ⇒ finite iterations OK
- Increased horizon ⇒ larger region of attraction

Experimental Results: Caltech Ducted Fan

NTG +
MPC +
CLF

Pitch
Control

dSPACE
RTOS

Multi-Vehicle Optimization-Based Control

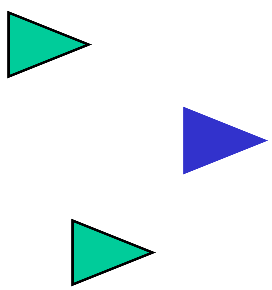
Assume we have real-time, finite horizon optimal control as a *primitive*

$$\left.\begin{aligned} u_{[t,t+\Delta T]} &= \arg\min \int_t^{t+T} L(x,u)\,dt + V(x(t+T)) \\ x_0 = x(t) &\quad x_f = x_d(t+T) \\ \dot{x} = f(x,u) &\quad g(x,u) \le 0 \end{aligned}\right\}$$

Choose f, g, L to represent the *coupling* between the various subsystems

Cooperation depends on how we model "rest of the world"

$$\ddot{q}_1 = 0 \quad |u_1| \le L$$

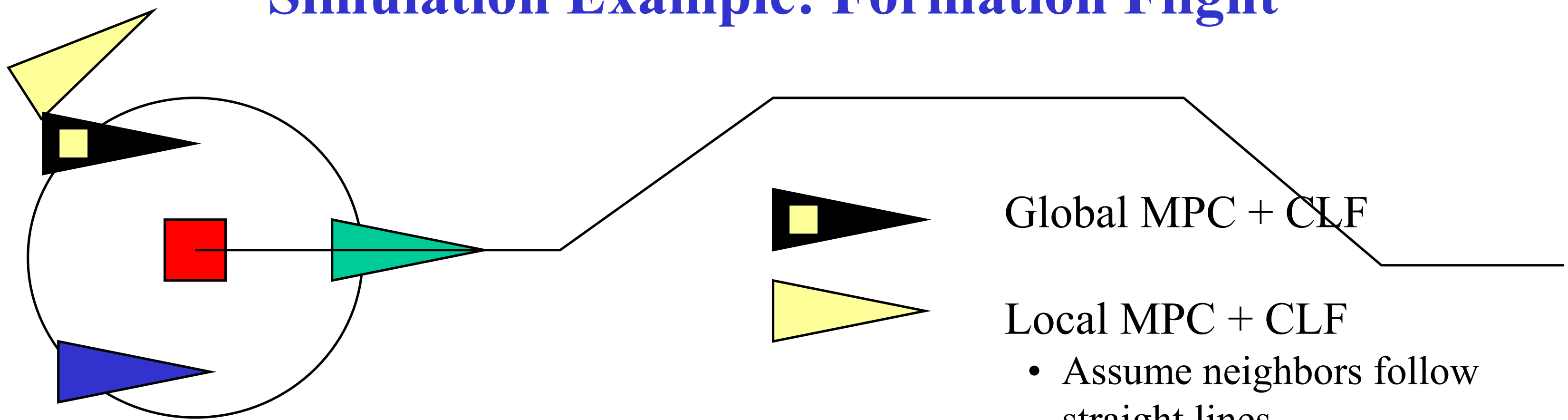


$$\boxed{\begin{aligned} u_{2,[t,t+\Delta T]} &= \arg\min \int_t^{t+T} L(q_1, x_2, q_3, u_2)\,dt + V \\ \dot{x}_2 = f(x_2, u_2) &\quad g(x_2, u_2) \le 0 \end{aligned}}$$

$$\ddot{q}_3 = 0 \quad |u_3| \le L$$

Reconfigurable based on condition, mission, environment

Simulation Example: Formation Flight



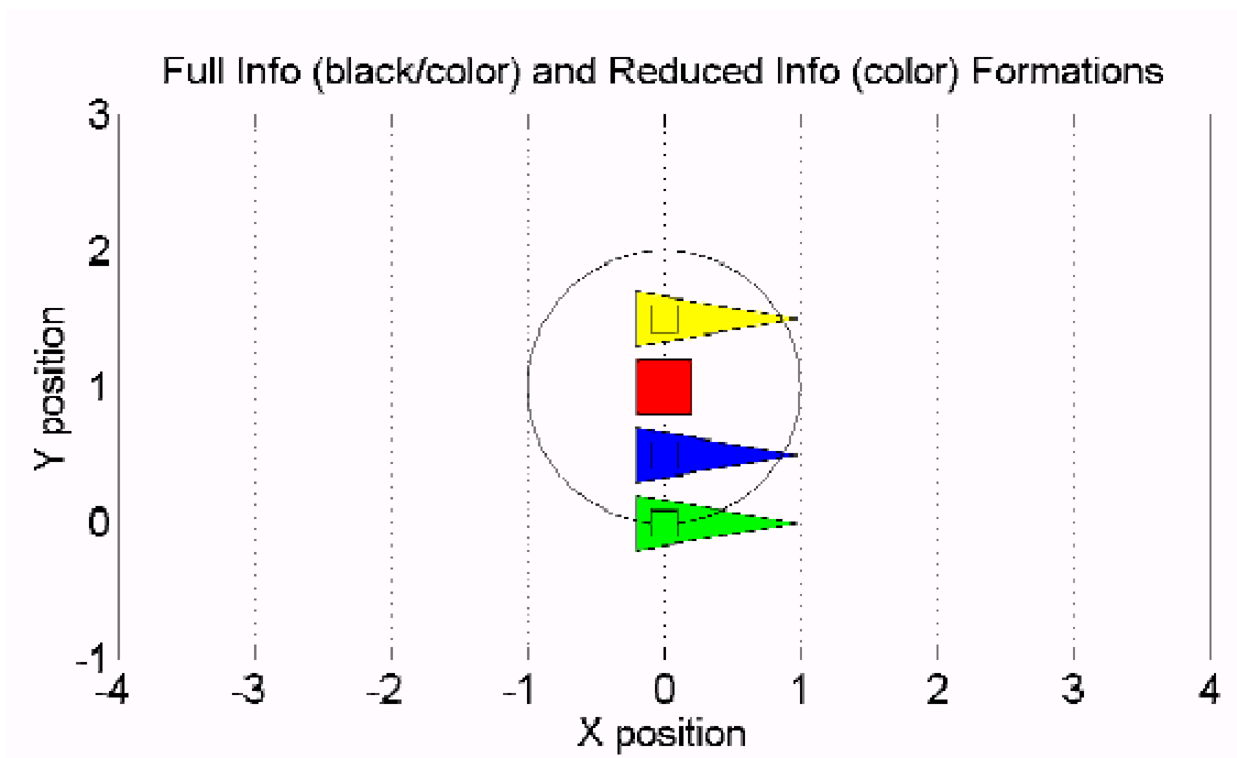
- Assume neighbors follow straight lines

Task:

- Maintain equal spacing of vehicles around circle
- Follow desired trajectory for center of mass

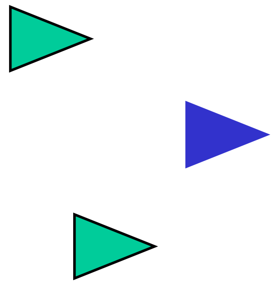
Parameters:

- Horizon: 2 sec
- Update: 0.5 sec
- **High damping**



Multi-Vehicle Optimization-Based Control

$$\ddot{q}_1 = 0 \quad |u_1| \leq L$$



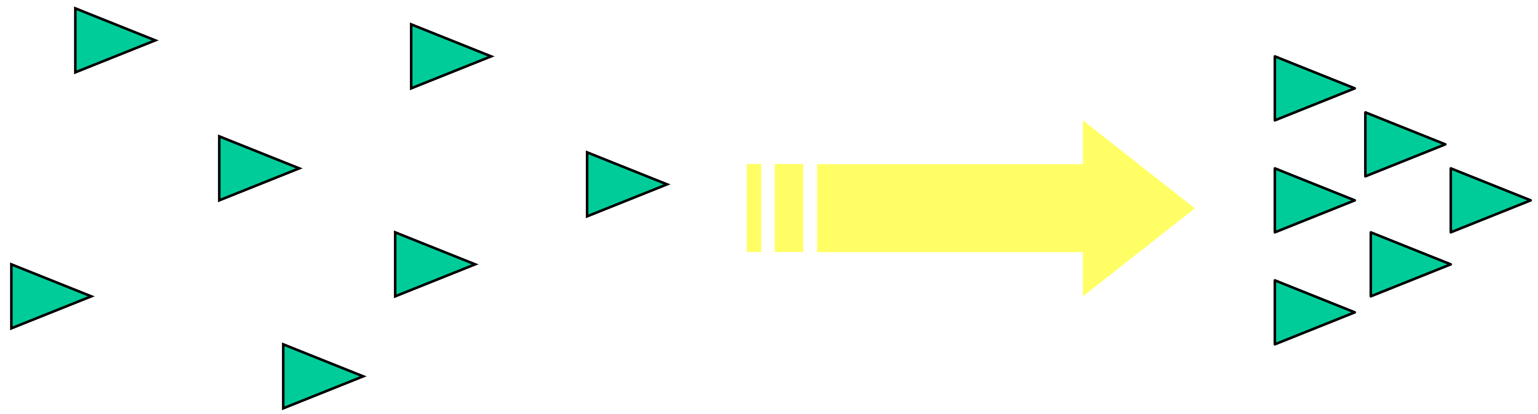
$$\boxed{\begin{array}{c} u_{2,[t,t+\Delta T]} = \arg\min \int_t^{t+T} L(q_1, x_2, q_3, u_2)dt + V \\ \dot{x}_2 = f(x_2, u_2) \quad g(x_2, u_2) \leq 0 \end{array}}$$

$$\ddot{q}_3 = 0 \quad |u_3| \leq L$$

Open Questions: "Low Level" Cooperation

- How do we coordinate motion between multiple vehicles?
- How do we aggregate cost functions into hierarchies?
- How do we provide redundancy and failure tolerance?
- How do we communicate between vehicles and how often?
- How do we insure scalability to large numbers of agents?
- How do we incorporate adversarial actions?

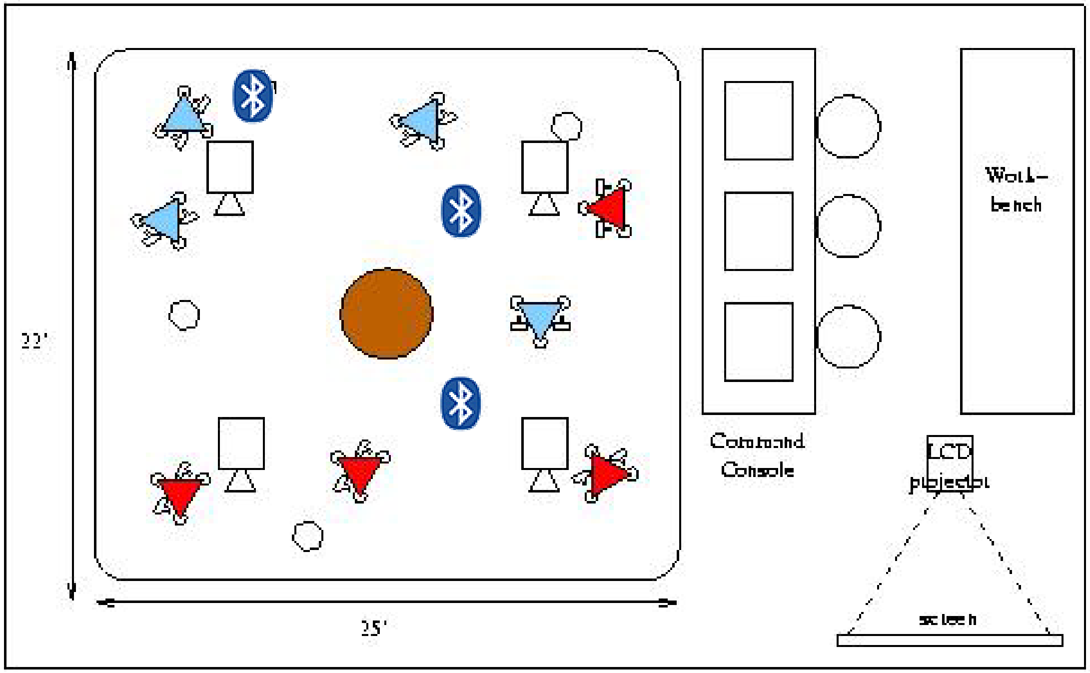
Higher Level Cooperation: Rejoin Exapmle

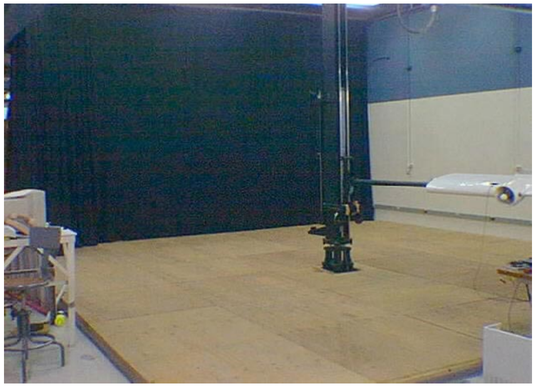


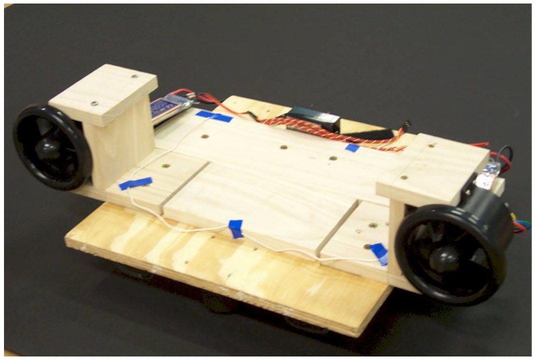
Open Questions: Higher Level Issues

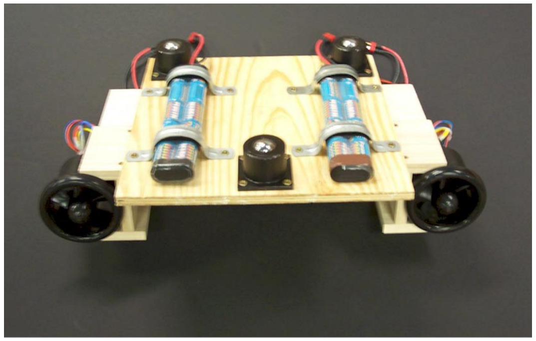
- How do we collectively agree to rejoin in a robust manner?
- How do we integrate “protocol stack” with “trajectory generation/tracking”
- How do we describe the “specification” of the task?
- How do we prove that solution (code) satisfies the specification?
- How do we prove “stability” of the solution?
- How do we verify and validate the solution?
- How do we insure all of this works in the presence of adversaries?

Multi-Vehicle Wireless Testbed for Integrated Control, Communications and Computation (DURIP)









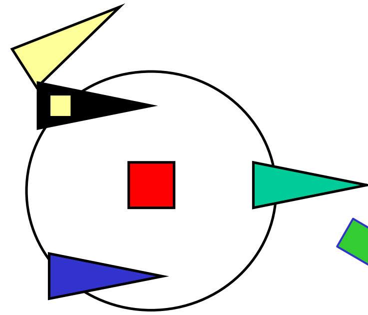
Testbed features

- Distributed computation on individual vehicles + command and control console
- Point to point, ad-hoc networking (bluetooth) + local area networking (802.11)
- Cooperative control in dynamic, uncertain, and adversarial environments

Research Plan and Integration: Cooperative Control in Dynamic, Uncertain and Adversarial Environments

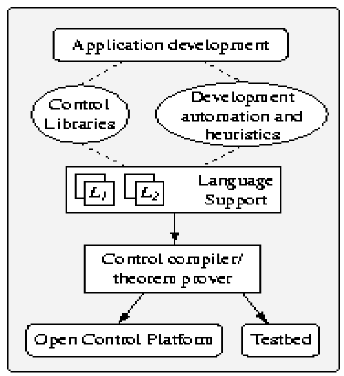
Optimization-Based Control

- Real-time model predictive control for online control customization: theory and software
- Online implementation on Caltech Ducted Fan



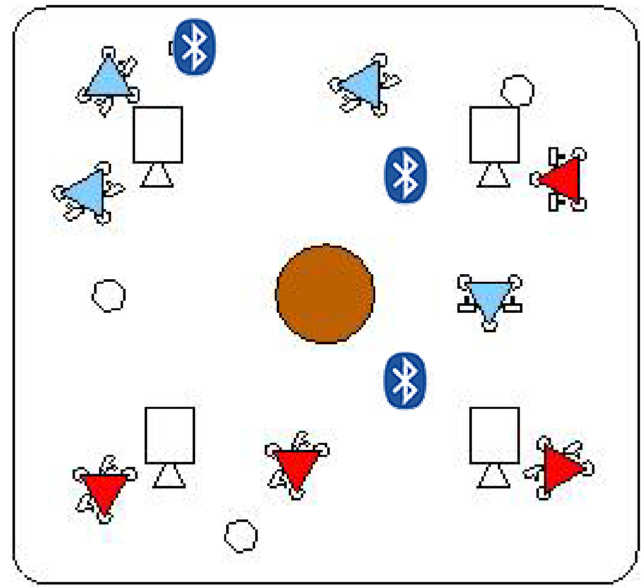
Cooperative Control

- Linked cost functions



Software Environments

- Logical programming environments for embedded control systems design



Multi-Vehicle Testbed (DURIP)

- Implementation on multi-vehicle, wireless testbed using Open Control Platform
- Bluetooth-based point to point communications with ad-hoc networking