Distributed Control of Multi-Vehicle Systems

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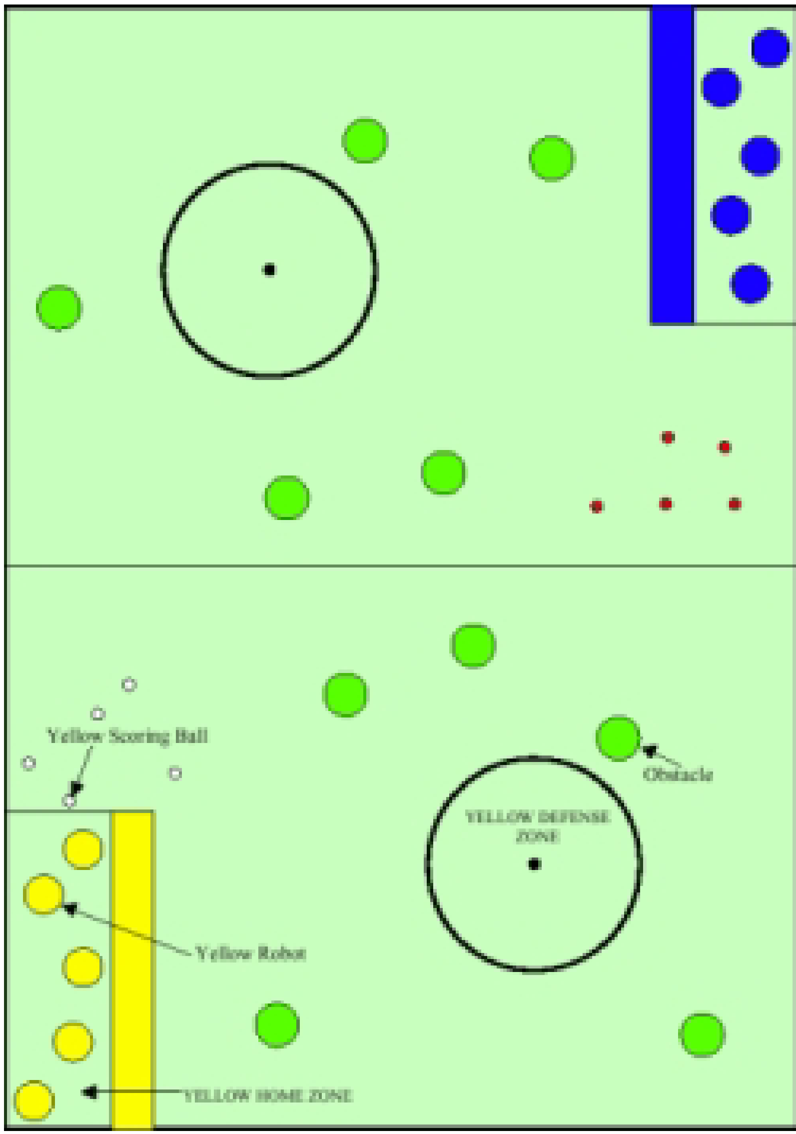
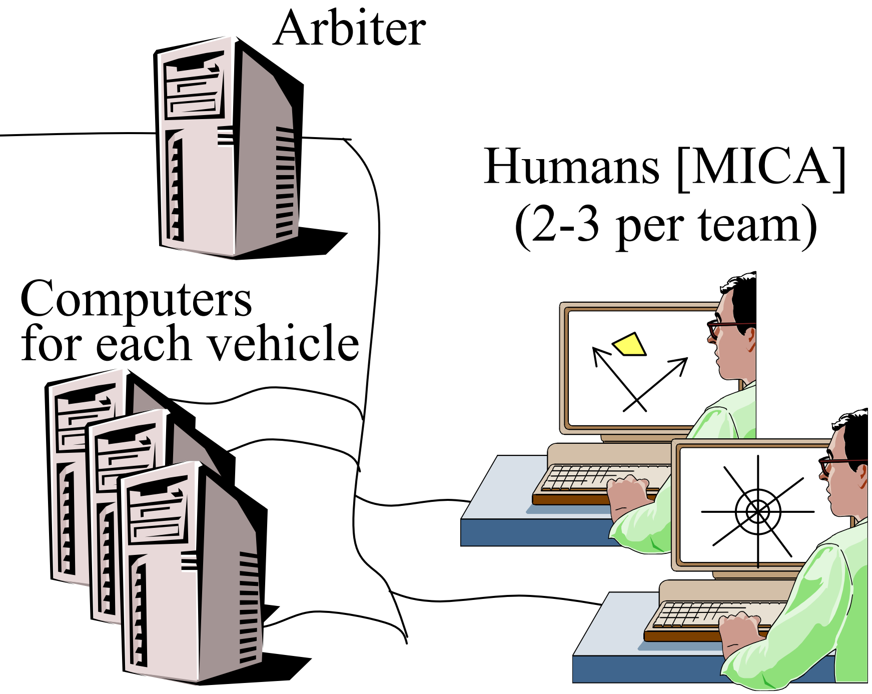
Outline

I. **Background and Motivation**
II. **Graph rigidity and distributed control of multi-vehicle formations**
III. **Computation and control (Hickey)**
IV. **Caltech multi-vehicle wireless testbed status**

Cooperative Control in Dynamic, Uncertain, Adversarial Environments





Team-based control for RoboFlag

- *Theory* for cooperative control of multi-vehicle systems in adversarial (competitive) environments
- Extend control approach to make use of tools from computer science (formal methods, protocol verification, etc)

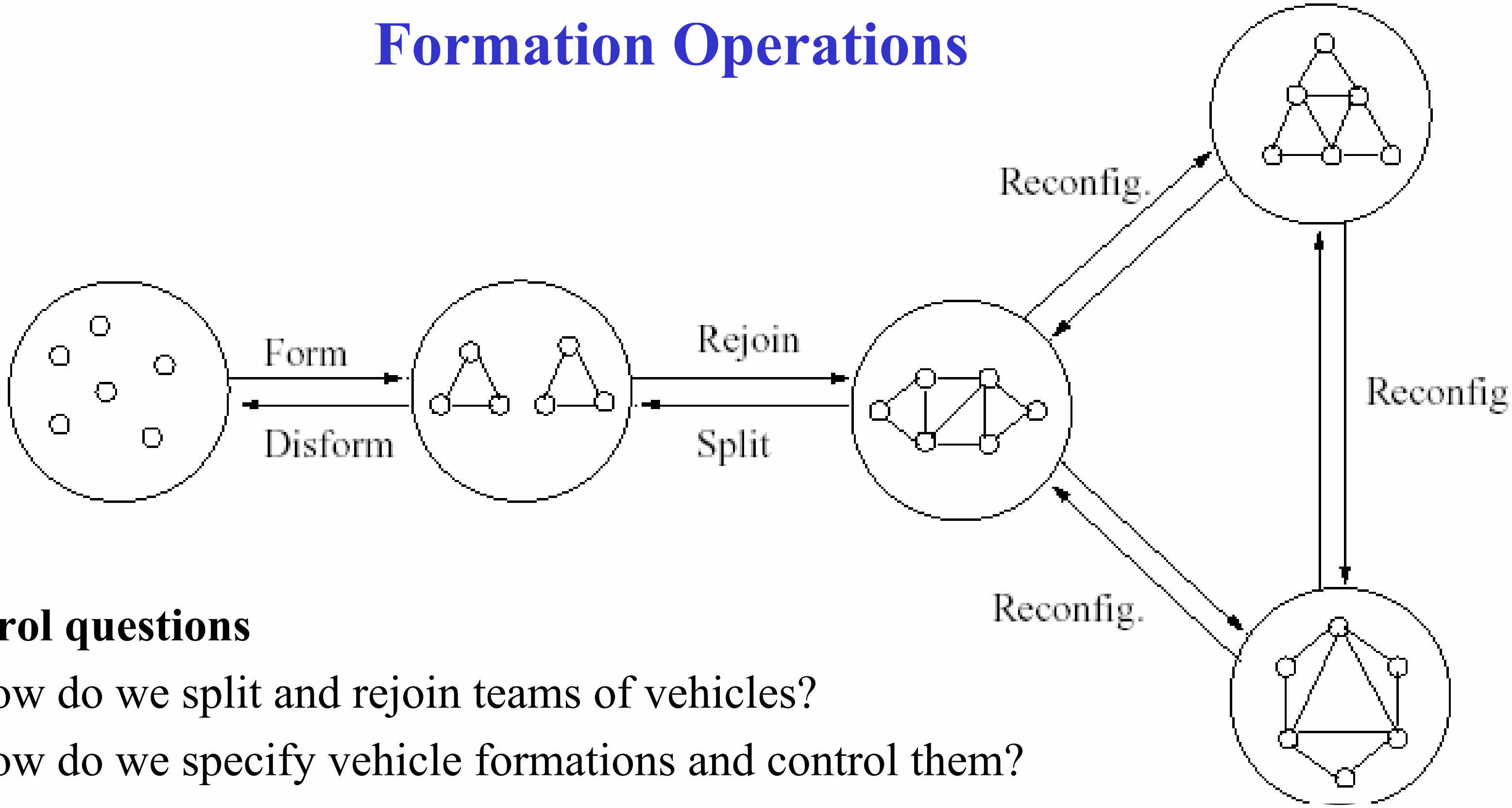
Graph Rigidity and Distributed Control of Multi-Vehicle Formations

Reza Olfati-Saber and Richard M. Murray

References

- Distributed Structural Stabilization and Tracking for Formations of Dynamic Multi-Agents, Reza Olfati-Saber and Richard M. Murray. Submitted, 2002 Conference on Decision and Control (CDC).
- Graph Rigidity and Distributed Formation Stabilization of Multi-Vehicle Systems, Reza Olfati-Saber and Richard M. Murray. Submitted, 2002 Conference on Decision and Control (CDC).
- Distributed Cooperative Control of Multiple Vehicle Formations Using Structural Potential Functions, Reza Olfati-Saber and Richard M. Murray. To appear, 2002 IFAC World Congress.

Formation Operations



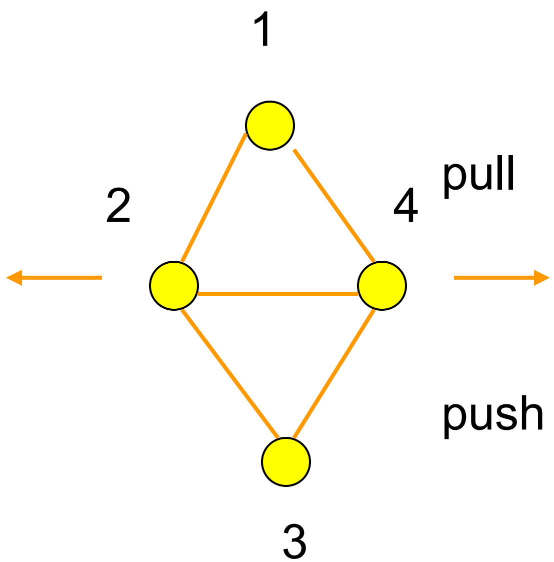
Control questions

- How do we split and rejoin teams of vehicles?
- How do we specify vehicle formations and control them?
- How do we reconfigure formations (shape and topology)

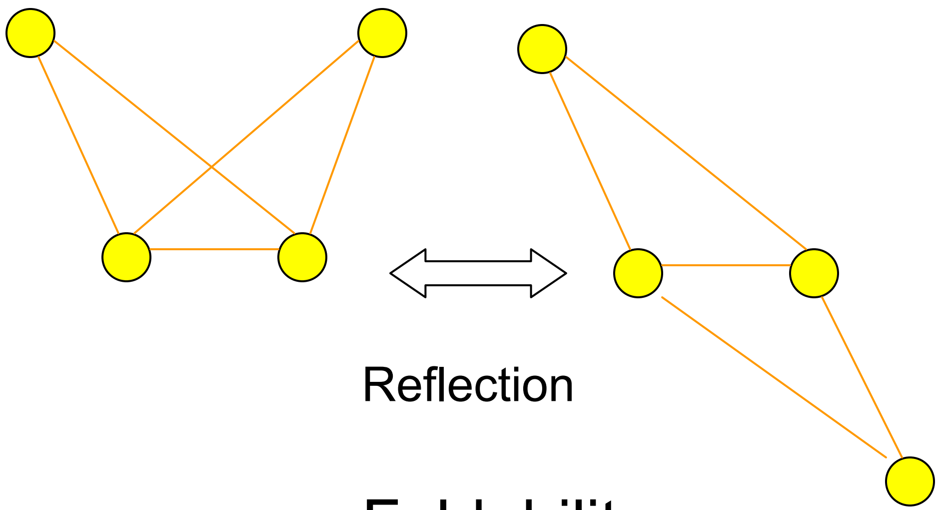
Initial approach: potential functions

- Provides natural mechanism for distributed control
- Can easily extend to optimization based control (with potential function as cost) $\Rightarrow$ take into account constraints, nonlinearity

Graph Rigidity and Foldability



Rigidity



Foldability

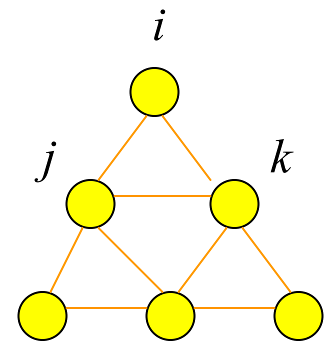
Definition A graph G is called a *rigid graph* iff there exists a subgraph H with n nodes and $2n$-3 edges of G such that

$$(q_j - q_i)^T \cdot (p_j - p_i) = 0 \quad \forall e_{ij} \in E_H$$
$$\Downarrow$$
$$(q_s - q_r)^T \cdot (p_s - p_r) = 0 \quad \forall r, s \in I, r \neq s$$

q_i = node position

p_i = node velocity

Task Specification



Unique formation representations

- Distance constraints (rigidity)
- Area-based constraints (foldability)

$$\Gamma = \left\{ q \in \mathbb{R}^{2n} : \begin{array}{c} \left\| q_j - q_i \right\| - d_{ij} = 0, \forall e_{ij} \in E, \\ (q_j - q_i) \otimes (q_k - q_i) - a_{ijk} = 0, \forall f_{ijk} \in F \end{array} \right\}$$

E = edges of G

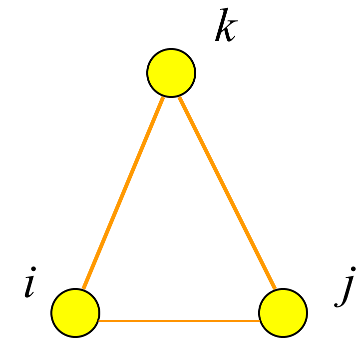
F = faces of G

Theorem (CDC '02): In $\mathbb{R}^2$, $2n$-3 distance-based and n-2 area-based algebraic constraints associated with "properly placed" edges and faces are required to uniquely specify a formation of n agents.

Formation Graph $G = (V, E, D, F, A)$

Q: how do we stabilize a formation with graph G?

Formation Potentials



Constraint deviation variables → formation potential

$$\eta_{ij} = \left\| q_j - q_i \right\| - d_{ij},$$

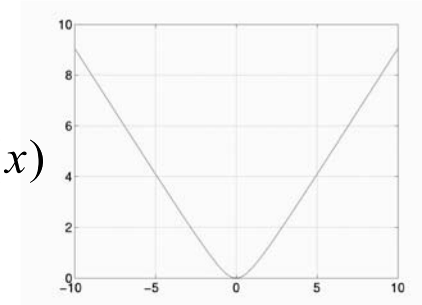
$$\delta_{ijk} = (q_j - q_i) \otimes (q_k - q_i) - a_{ijk},$$

$$V(q) = \sum_{e_{ij} \in E} \phi(\eta_{ij}) + \sum_{f_{ijk} \in F} \phi(\delta_{ijk})$$

Potential function properties, example

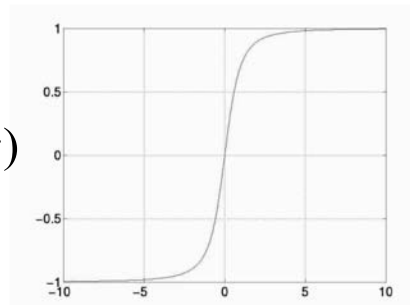
$$\begin{cases} \phi(x) > 0, \forall x \neq 0 \\ \phi(0) = 0 \end{cases}$$

$\phi(x)$



$$\phi(x) = \sqrt{1 + x^2} - 1 \rightarrow f(x) = \phi'(x) = \frac{x}{\sqrt{1 + x^2}}$$

$f(x)$



Example: Triangle

$$V(q) = \phi(\eta_{12}) + \phi(\eta_{23}) + \phi(\eta_{32}) + \phi(\delta_{123})$$

Control Results

Problem statement: Given $H(q,p)=\frac{1}{2}\sum_{i\in I}\|p_i\|^2+V(q)$ find u such that

$$\frac{dH}{dt}\le 0 \quad \text{and} \quad \lim_{t\to+\infty} H(q,p)=0$$

Distributed control (IFAC '02)

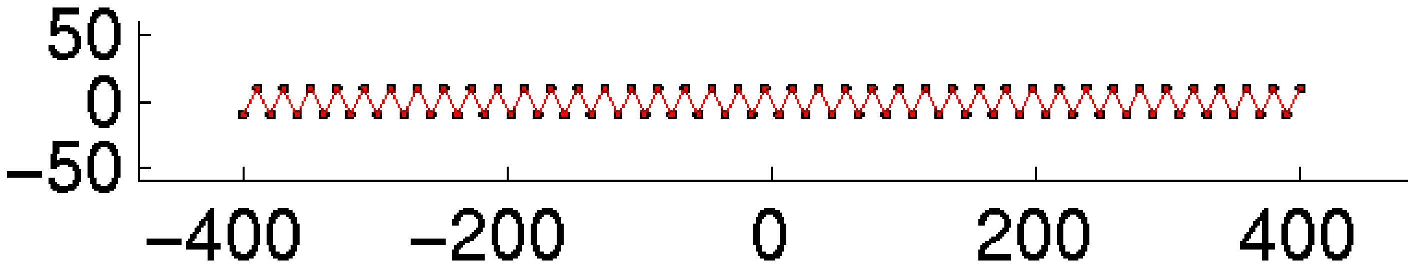
$$u_i=\sum_{j\in J_i} f(\|q_j-q_i\|-d_{ij})\cdot\vec{n}_{ij}-c_d p_i$$

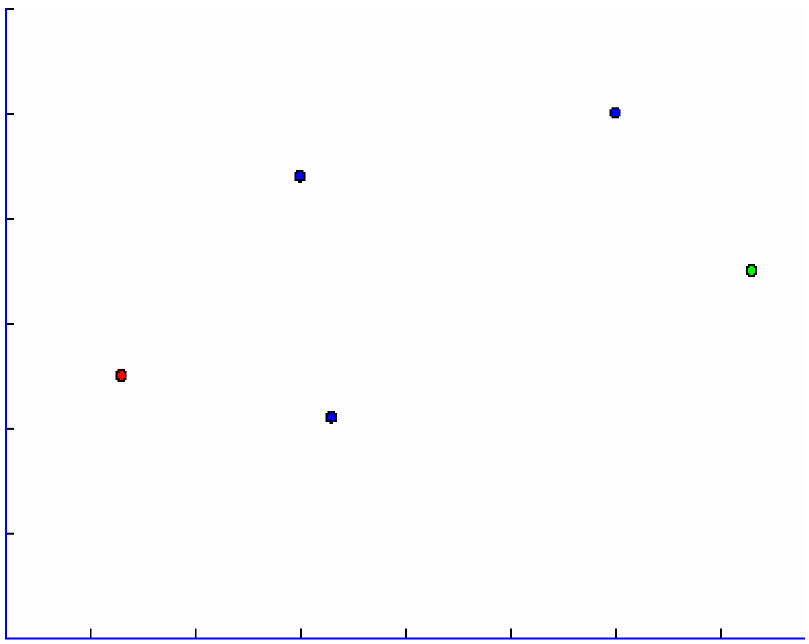
Theorem: If each vehicle applies the control input in (*), then the trajectory of the group of vehicles locally asymptotically converges to the desired formation in a collision-free manner.

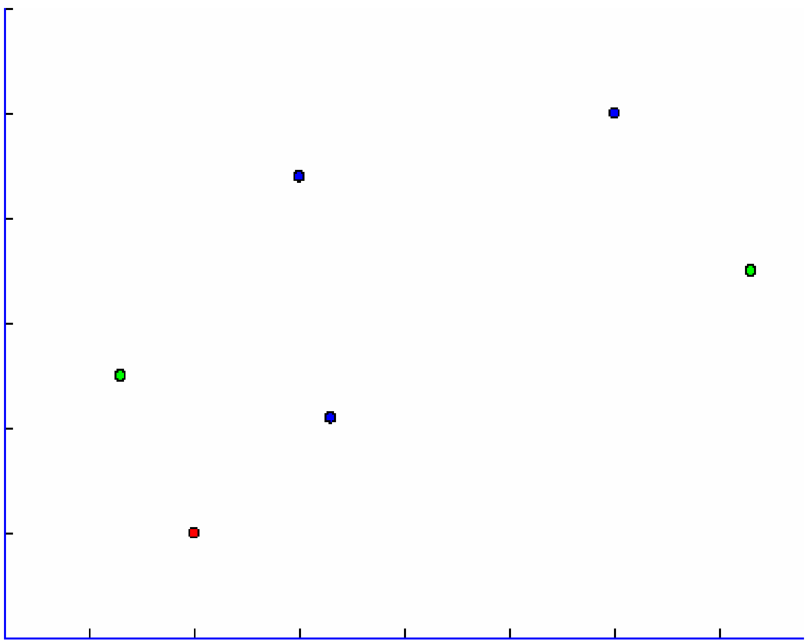
Optimization based control

$$u^*=\underset{u\in\mathcal{U}}{\arg}\underset{\ddot{q}_i=u_i}{\min} J=\int_0^T H(q,p)dt+H(q(T),p(T))$$

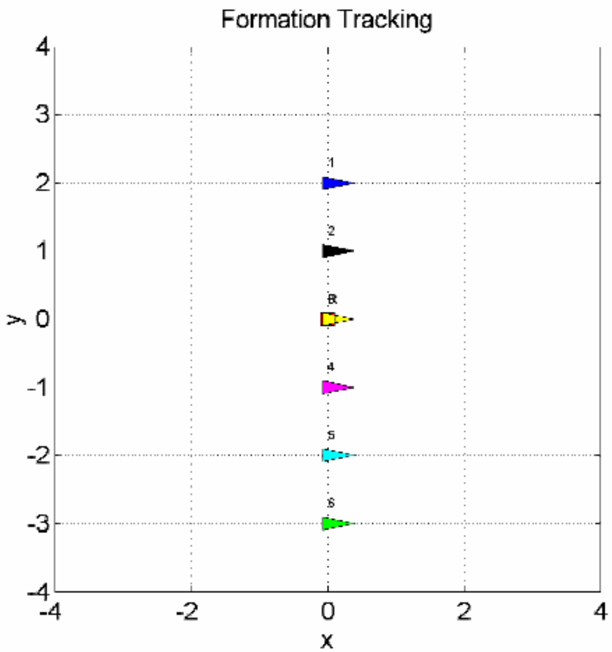
Distributed Control of Multi-Vehicle Formations

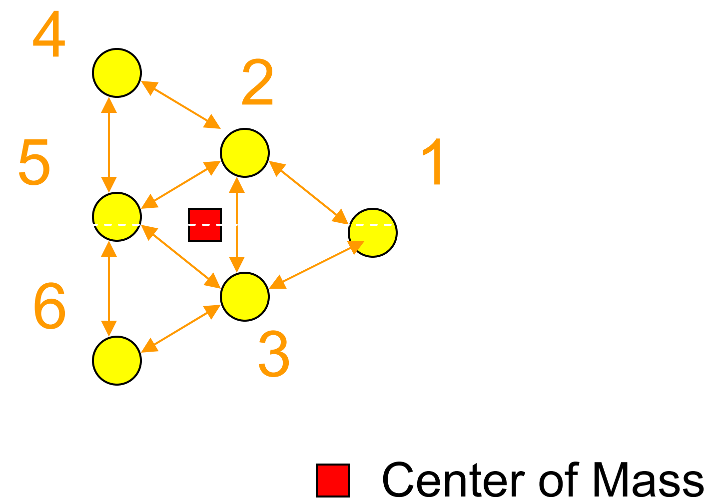


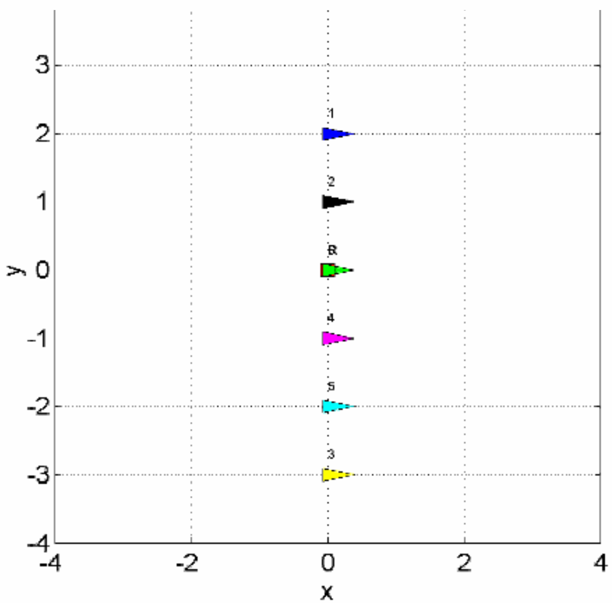


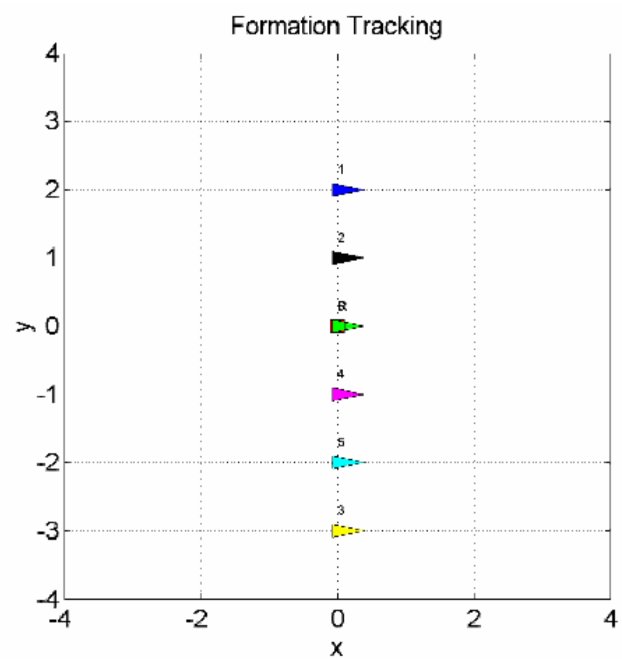


Optimization-Based Tracking









Computation and Control

Adam Granicz and Jason Hickey

References

- Formal Design Environments, Adam Granicz, Brian Aydemir, and Jason Hickey. Thereom Proving and Higher Order Logic (TPHOL), 2002.
- Adam Granicz and Jason Hickey. Phobos: An Approach to Domain Specific Compilers, HICSS Workshop on Domain Specific Languages

Caltech Multi-Vehicle Wireless Testbed

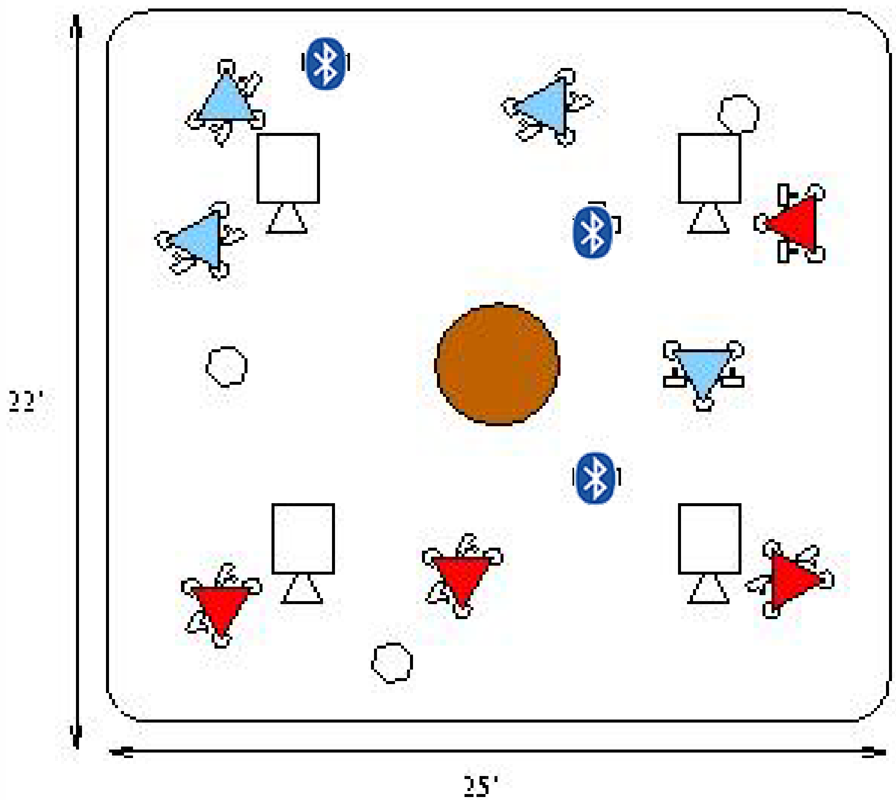
Richard M. Murray   Jason J. Hickey   Steven Low

Lars Cremean   William Dunbar   David van Gogh   Eric Klavins
Jason Meltzer   Abhishek Tiwari   Steve Waydo

References

- The Caltech Multi-Vehicle Wireless Testbed, Lars Cremean, William Dunbar, David van Gogh, Jason Hickey, Eric Klavins, Jason Meltzer, Richard M. Murray. Submitted, 2002 Conference on Decision and Control (CDC)
- http://www.cds.caltech.edu/~mvwt

Multi-Vehicle Wireless Testbed for Integrated Control, Communications and Computation (DURIP)

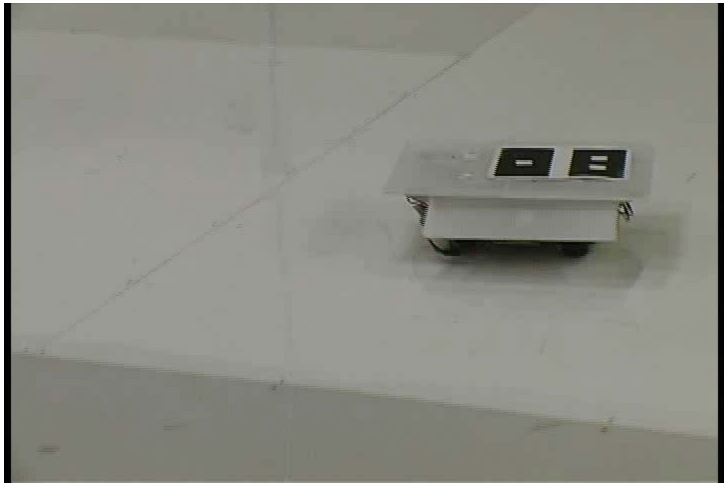


Testbed features

- Distributed computation on vehicles + command and control console
- Point to point networking (bluetooth) + local area networking (802.11)
- Overhead vision system provides global position data (LPS)

Results to Date

Manual Control (9 Nov 01)

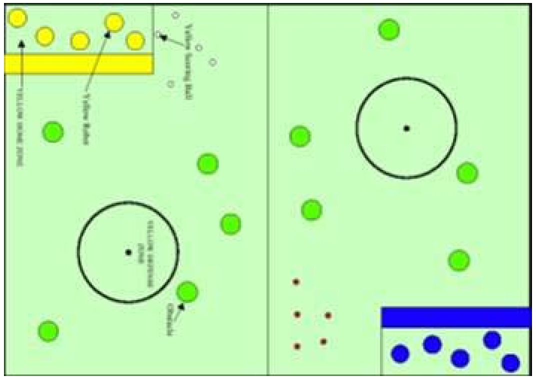
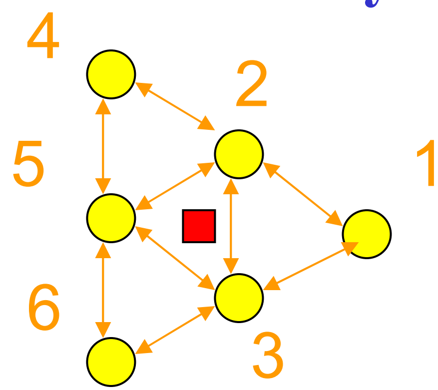


Classical Control (7 May 02)

Trajectory Tracking (28 May 02)

Leader Follower (28 May 02)

Summary

Motivation: Cooperative Control in Dynamic, Uncertain, Adversarial Environments

- RoboFlag as driver for theory of teams, cooperation, distributed control, etc
- Combine ideas from control and computer science → higher levels of decision making

Current work

- Graph rigidity and distributed control: strong framework for future results
- Logical programming environments: basic infrastructure and theory
- Caltech multi-vehicle wireless testbed: experimental platform of testing ideas

Next steps

- Stronger theory for mixed logical/continuous operations (split/rejoin)
- Beat Cornell at RoboFlag through superior theory, strategy, and tactics