Theory and Experiments on the Generation of Spontaneous Emission Using a Plasma Wave Undulator: A Progress Report

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ABSTRACT

We are studying the feasibility of using relativistically moving plasma waves as short wavelength undulators for possible FEL and Compton scattering applications at UCLA. The remarkable property of such waves is that the wiggler parameter $a_w = eA/mc^2$ can be on the order 0.1 while their wavelength λ_w can be submillimeter. Such waves can be excited by either an intense electron bunch going through a plasma (plasma wake field) or a short but intense laser pulse going through the plasma (laser wake field).

A variation of the laser wake field scheme is the plasma beat wave excitation. Here a moderately intense laser pulse containing two frequencies excites the plasma wave resonantly. Using a laser pulse containing 10.27 μm and 9.6 μm lines of the CO_2 laser that is approximately 400 ps (FWHM) and 200 GW of power, we were able to measure a_w times the length product of 0.013 cm in our experiments. If a length of 0.75 cm is assumed, this implies an a_w of 0.17 for a $\lambda_w \sim 156$ μm. Injection of an electron beam across such a plasma wave proved not to be feasible in these experiments, because the θ-pinch plasma source contained significant trapped magnetic fields. We are currently developing a field free plasma source which will permit transverse electron injection.

On the theoretical front, the electron trajectories in a relativistic plasma wave undulator have been simulated using a 3D model which includes emittance effects. We calculate the resulting radiation pattern produced by the electrons. We also analyze the scattered electron energies and spatial distributions, which may be useful as diagnostics in an experiment.

1. INTRODUCTION

This paper presents a progress report on the theoretical and experimental work being done at UCLA and at Spectra Technology Inc. on the generation of spontaneous emission using relativistic plasma waves as undulators/wigglers. The words undulator and wiggler will be used interchangeably since the normalized strength of the field is $0.1 < a_w < 1$ and $10 < N < 100$. Here $a_w = eE/m\omega_{pc}c = eA/mc^2$ and N is the number of wiggle periods. The work has been motivated by the potential application of such a wiggler to the generation of spontaneously emitted x-ray photons. As it is well known, a general figure of merit for a wiggler is that the product of the field strength and the wavelength should ideally be on the order of 10 kG-cm. One method of achieving this at millimeter and still smaller wiggler wavelengths is the use of the oscillating electric fields of a relativistic plasma wave.[1] The remarkable property of such waves is that irrespective of their wavelength, at the "wavebreaking limit" the product of the longitudinal electric field and the apparent wiggler wavelength is constant and this product is equivalent in strength to that of a 10 kG-cm static magnetic wiggler. Unfortunately such a wiggler is probably not suited for FEL applications because of the interaction of the electron beam with the plasma and also because of the short duration (100 ps) over which such a wave remains coherent.[2] Accordingly our effort is aimed at studying the spontaneous emission emitted by a single electron bunch as it traverses the wiggler.

In section 2, the basic characteristics of the radiation from such a plasma wave wiggler are identified. In section 3, we present the numerical work which studies in detail the wave-particle interaction physics. In section 4, we discuss the status of our experiments which, until now, have concentrated on the characterization of such a plasma wave undulator with some preliminary studies of electron injection in such waves.

2. CHARACTERISTICS OF RADIATION FROM A PLASMA UNDULATOR[3]

The simplest geometry of the plasma wiggler is shown in Fig. 1. In this geometry the laser beams that excite the plasma wiggler and the electron beam are transverse to one another. Thus, it is the longitudinal field of the plasma wave that is used to "wiggle" the beam. In such a wiggler the number of photons per electron is given by $0.0153\ Na_w^2$. Here we assume that a_w is constant over the N periods. The radiated photon frequency on axis is given by $\omega = 2\gamma^2\omega_p$. We define the brightness of the photon beam as

$$\text{Brightness} = \frac{\begin{bmatrix}\text{total number}\\ \text{of electrons}\end{bmatrix}\begin{bmatrix}\text{fraction of electrons}\\ \text{participating}\end{bmatrix}\begin{bmatrix}\text{photons emitted}\\ \text{per electrons}\end{bmatrix}\times 10^{-3}}{\begin{bmatrix}\text{emission solid}\\ \text{angle}\end{bmatrix}\begin{bmatrix}\text{emission}\\ \text{area}\end{bmatrix}\begin{bmatrix}\text{fractional}\\ \text{bandwidth}\end{bmatrix}(\text{pulselength})} \tag{1}$$

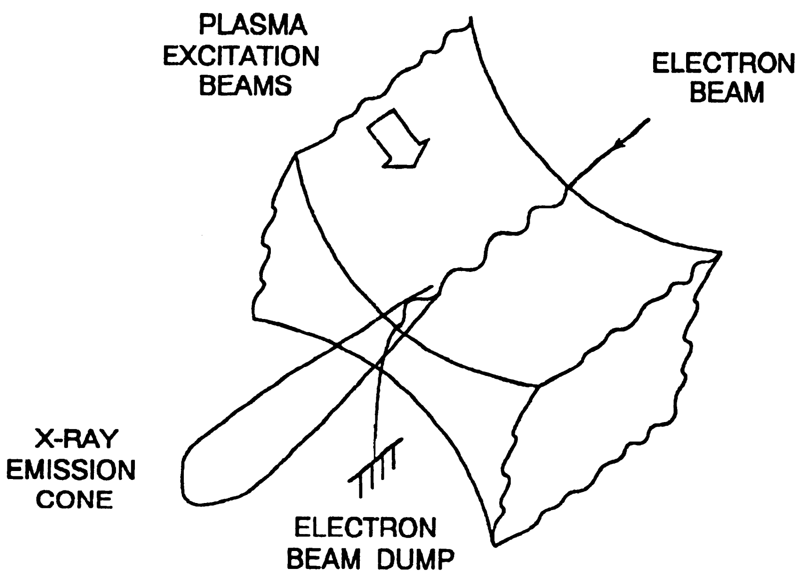

Figure 1. Geometry of plasma wiggler.

Electron beam		
Charge	q	1 nC
Normalized emittance	ε_N	0.01 cm rad
Pulse length	τ	1 ps
γ		223
Wiggler Parameters		
wiggler strength	a_w	0.1
N		100
wiggler wavelength	λ_w	100 μm
wiggler radius	r_{plasma}	500 μm
radiated wavelength	λ_r	10 Å

Table 1. Typical Parameters for Plasma Wiggler Brightness Calculation

We can estimate line brightness for a state-of-the-art electron beam as it traverses the plasma undulator and compare it with existing or under construction synchrotron sources. Typical parameters are given in Table 1.

It can be shown that for the above parameters the electron beam "underfills" the wiggler and all the electrons participate.[3] The brightness expression can then be re-written, assuming N = 100 and $a_w = 0.1$ as,

$$\text{Brightness} = \frac{3.83 \times 10^{17}\, q\, [\text{nC}]\, \gamma^2}{\varepsilon_N^2\, \tau\, [\text{psec}]} \quad \frac{\text{photons}}{(\text{rad})^2\, (\text{cm})^2 (0.1\%\ \text{BW})(\text{sec})} \tag{2}$$

Fig. 2 shows how the plasma wave wiggler compares with existing and proposed synchrotron radiation devices[4] in the 10 eV–10 keV range. It should be noted that we have assumed a 1 nC charge in 1 ps (i.e., I_{peak} = 1kA) which is far greater than the circulating beam currents in the storage rings. One can see that in the best scenario the plasma wave undulators will be comparable in performance with the existing machines but not competitive with those under construction.

3. NUMERICAL STUDIES

We are carrying out detailed numerical studies of electron motion and radiation emission from plasma wave undulators. In the model the peak amplitude of the plasma wave is uniform in the direction of the plasma wave phase velocity $v_z \cong c$, and Gaussian in the directions (x,y) perpendicular to z. Therefore, the electrons are acted upon by both longitudinal and transverse fields of the plasma wave. The space charge repulsion among the electrons is considered negligible and $a_w \leq 0.15$. The maximum longitudinal field scales as $\sqrt{n}$ V/cm, where n is the plasma density in cm^{-3}. The longitudinal and transverse plasma wave fields in two dimensions (E_z and E_r) which act on the electrons are given by [5]

$$E_z(r,z,t) = +\frac{mc\omega_p\alpha}{e}\, e^{\left(-2r^2/R^2\right)} \left[\omega_p t \cos(k_p z - \omega_p t) - \cos(k_p z)\sin(\omega_p t)\right]$$

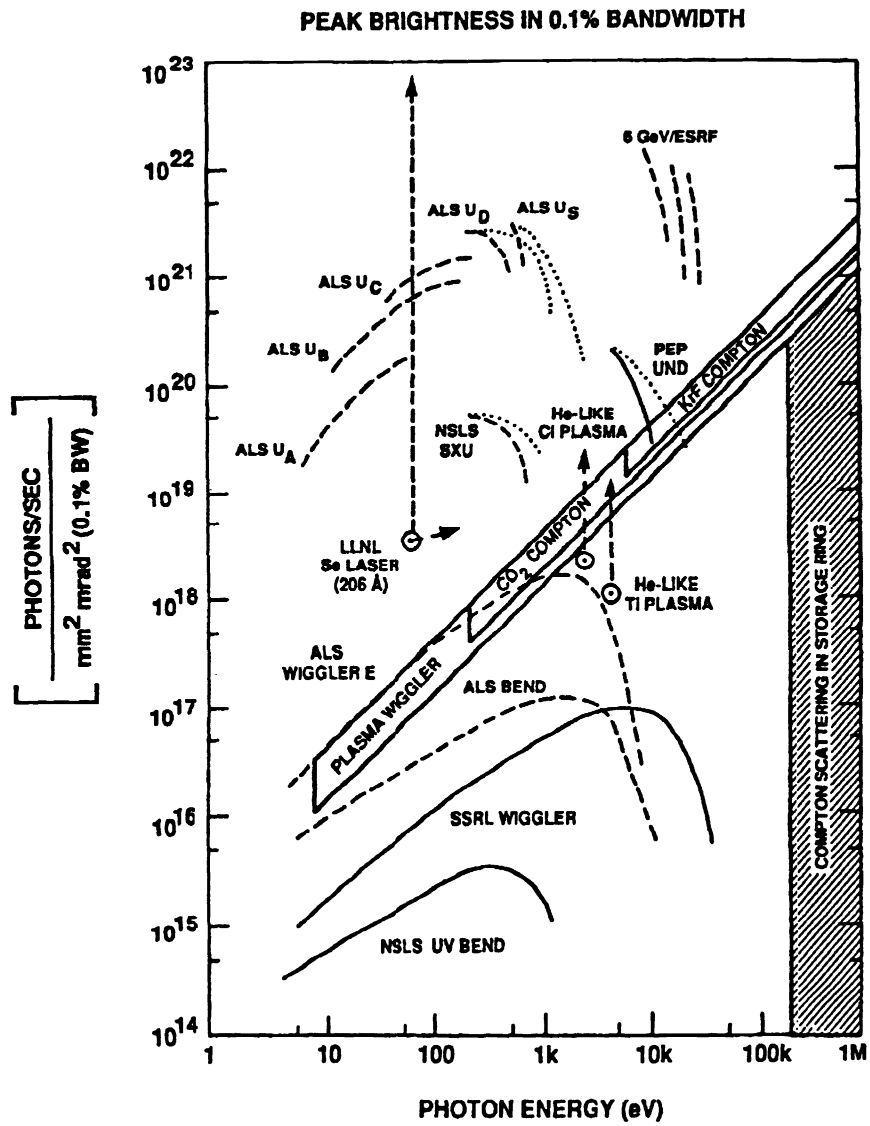

Figure 2. Peak brightness with comparison with several existing and planned storage rings.

and

$$E_r(r,z,t) = -\frac{4mc\omega_p\alpha r}{ek_pR^2}\, e^{\left(-2r^2/R^2\right)} \left\{ \omega_p t\sin(k_p z - \omega_p t) - 2[1 - \cos(\omega_p t)] - \sin(k_p t)\sin(\omega_p t) \right\}$$

where r, z = radial and longitudinal position of electron
m, e = mass and charge of electron, respectively
k_p, ω_p = plasma wave number and plasma frequency, respectively
R = laser beam mean radius.
$\alpha = \left(\frac{eE}{2m\omega_o c}\right)^2$.
E = maximum wave breaking field of the plasma wave
ω_o = laser pump frequency
t = time

The relativistic electron equation of motion solved in the simulation is:

$$\frac{d\bar{P}}{dt} = \frac{d(\gamma m\bar{v})}{dt} = e(\bar{E}_z + \bar{E}_r) \quad (3)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\bar{P}$ = electron total momentum
$\bar{v}$ = electron total velocity = $\bar{v}_z + \bar{v}_r$
γ = electron relativistic Lorentz factor.

in addition

$$\bar{v} = \frac{\bar{P}}{m\sqrt{1 + \frac{P^2}{m^2c^2}}}$$

$$\frac{dr}{dt} = v_r, \qquad \frac{dz}{dt} = v_z \quad .$$

In the three dimensional simulations E_r is replaced by:

$$E_x(r,z,t) = \frac{x}{r} E_r(r,z,t)$$

and

$$E_y(r,z,t) = \frac{y}{r} E_r(r,z,t)$$

where x = x displacement of electron from axis of wave
y = y displacement of electron from axis of wave

and

$$\frac{dx}{dt} = v_x, \qquad \frac{dy}{dt} = v_y$$

We numerically solved equation (3) for the electrons' momenta and coordinates.

3.1. Electron Trajectories

As an example of the two dimensional trajectory plots, Fig. 3 shows trajectories in two dimensions for three electrons injected perpendicular to the plasma wave. The electrons are injected having different initial energies, γ_o = 4, 8 and 16. The plasma wave (not shown) moves from bottom to top in the figure, is centered at 0.0 and has a Gaussian half width corresponding to 50 λ_p (0.5 cm). The electrons move from left to right and are injected parallel to each other at an initial radius of $-150\ \lambda_p$ (-1.5 cm) and drift for 300 plasma wave periods. The plasma wave parameters are: $\gamma_{ph} = 9.7$ (corresponding to the 9.6 and 10.6 µm beatwave combination), $a_w = 10\%$, plasma wave width = 50 λ_p and $\gamma_o = 4$, where

λ_p = wavelength of plasma wave
γ_o = relativistic Lorentz factor for the initial electron injection energy
γ_{ph} = relativistic factor for the phase velocity of the plasma wave

$$= \frac{1}{\sqrt{1 - \frac{v_{ph}^2}{c^2}}} = \frac{\omega_o}{\omega_p}$$

where

$$\frac{v_{ph}^2}{c^2} = 1 - \frac{\omega_p^2}{\omega_o^2}$$

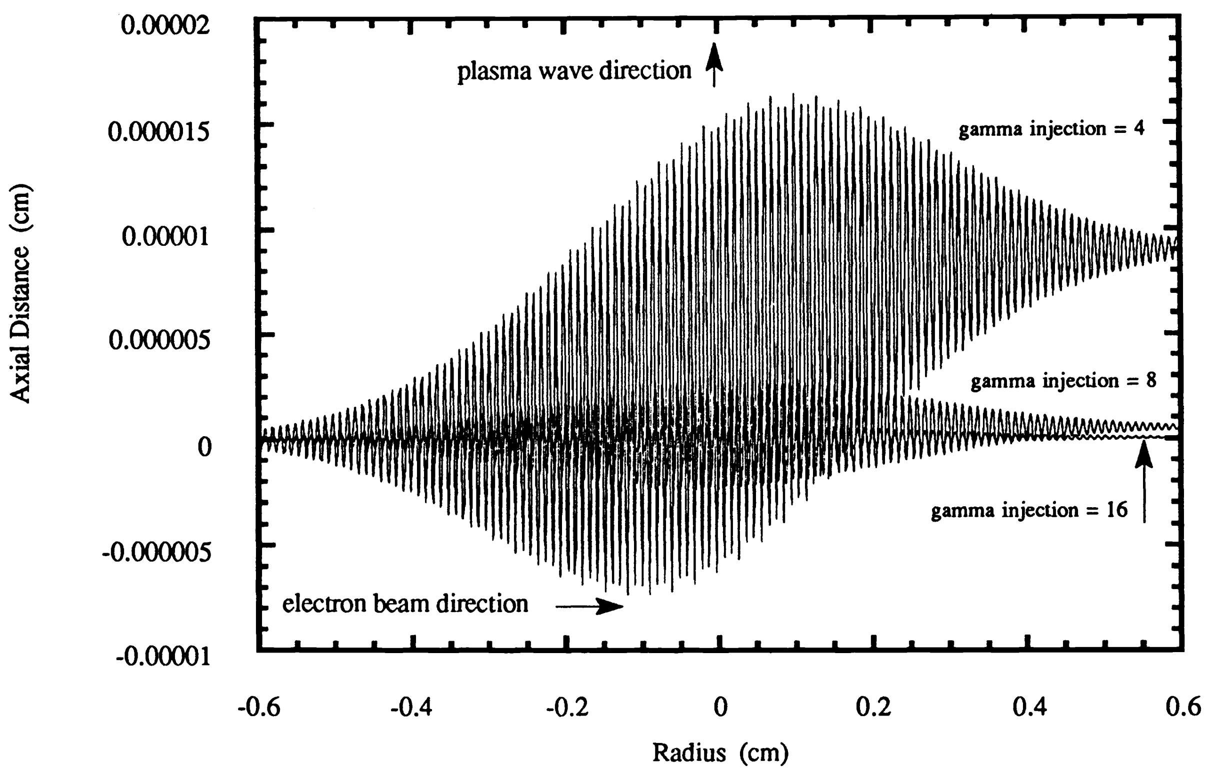

Figure 3. Trajectories of electrons injected transverse to a plasma wave. Electron initial energies are γ_o = 4, 8 and 16.

The major features of the electron trajectories[6] are a small drift in the direction of the plasma wave velocity and undulations while inside the plasma wave, both due to the plasma wave's longitudinal electric field. The electrons always drift in the direction of the plasma wave and the undulations result in radiation emission. We varied several electron and plasma wave parameters to understand their effects on the undulations and trajectories. We found that the amplitude of the undulations increased with the plasma wave amplitude, but inversely with the electron injection energy. The undulator wavelength increased with the plasma wave wavelength. The electron drift increased with plasma wave amplitude and plasma wave width, but inversely with electron injection energy. Some trajectories were deflected through a small angle after passing through the wave. This angle depended on the phase at which the electrons were injected in the plasma wave, and could be positive or negative. The deflection angle increased with plasma wave amplitude and plasma wave width, but inversely with electron injection energy.

3.2. Electron radiation

The radiation pattern of the undulating electrons is calculated by following the general approach given by Jackson.[7] We show as an example a radiation pattern of a single electron, $\gamma = 4$, injected perpendicularly into a plasma wave with $a_w = 0.2–0.6$, $\gamma_{ph} = 9.7$ and N = 50. (Fig. 4). The plasma wave moves from the bottom to the top while the electron moves from the left to the right of the figure. Once again the electron energy used is the same as what is expected in an experiment now underway at UCLA. As expected one observes the characteristic forward-peaked emission in a cone angle $1/\gamma$. We are using this model to calculate the angular and spectral distribution of radiation in our experiment.

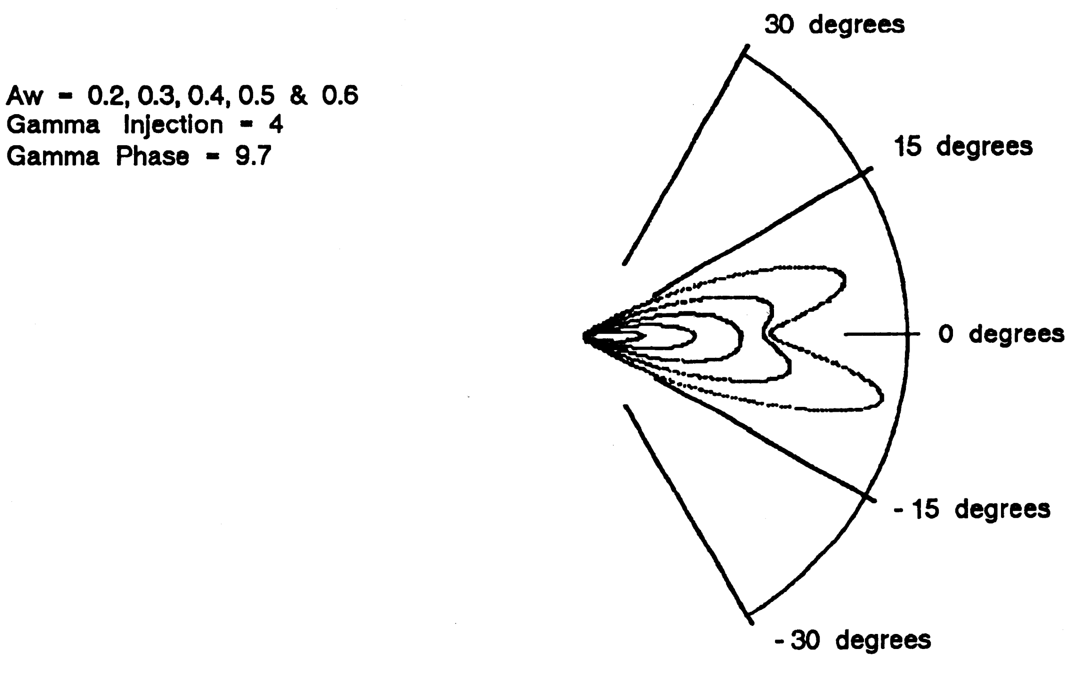

Figure 4. Radiation patterns for an electron drifting perpendicularly through plasma waves having a_w = 0.2, 0.3, 0.4, 0.5 and 0.6.

4. EXPERIMENTS ON EXCITATION OF A PLASMA UNDULATOR

The plasma wave undulator experiment at UCLA is designed to a) study the feasibility of generating very intense plasma waves with sufficient number of wiggle periods; b) determine how long such a structure remains coherent; and (c) study the radiation emitted by an injected electron bunch. Although most of the discussion hitherto has been on the transverse injection of the electron beam into the plasma wave, it is possible to utilize the radial field rather than the longitudinal field to "wiggle" the electron in which case the electrons are injected counterstreaming to the plasma wave.[10] The latter geometry is in principle, simpler because it does not require a wide plasma wave but rather a long plasma wave. The latter can be generated using simple spherical focusing optics for the laser beams. There are however some complications when using this counterstreaming scheme. In order that the electron beam is both focused and wiggled as it traverses the plasma wave, it (the wave) has to have a radial field profile that is maximum on axis.[10] Such a profile can be generated by beating two laser beams with different transverse mode structures, one operating in the TEM_{11} and the other in the TEM_{00} mode. Our experiments to date have been carried out with both laser beams in the TEM_{00} mode so that the feasibility of generating simultaneously large a_w and short λ_w can be demonstrated.

4.1. Experimental arrangement

There are four major components to our experiment: (1) the CO_2 laser which drives the plasma wave undulator; (2) the plasma source in which the undulator is formed; (3) the electron linac for injecting the electrons into the plasma wave; and (4) the radiation detection spectrograph and streak camera. These four components of the experiment are shown schematically in Fig. 5.

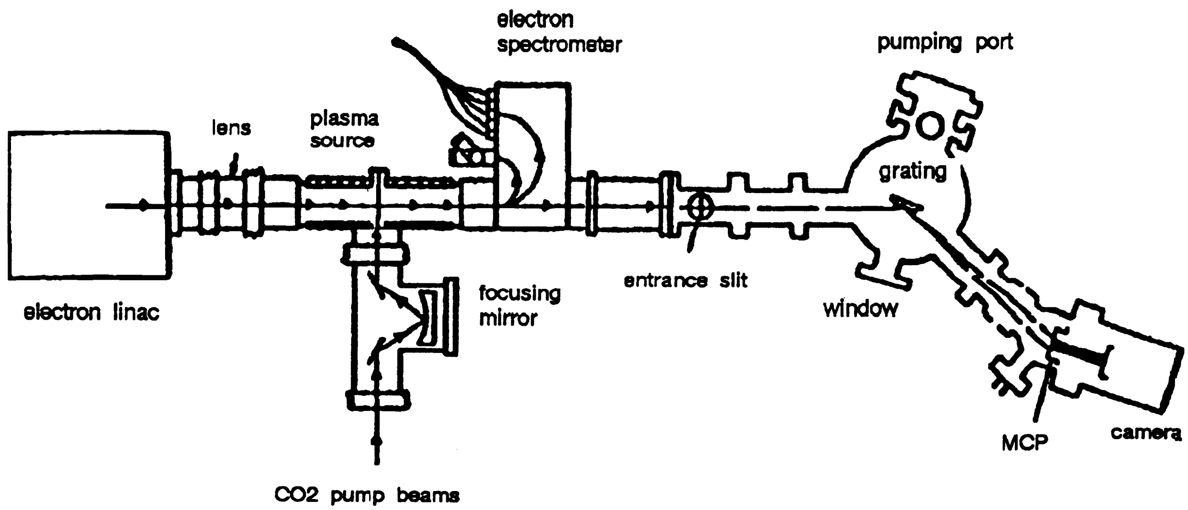

Figure 5.

(1) The laser system delivers 200 GW pulses containing two wavelengths of light, 10.27 μm and 9.56 μm. The pulse duration is about 400 ps. This system has been described in detail elsewhere.[8] (2) The plasma source is a theta-pinch, chosen for its ability to provide pre-formed, fully-ionized plasmas with requisite spatial uniformity. Beat wave excitation of a plasma wave is a resonant process, requiring hundreds of beat wave periods to build up the wave and an extremely uniform plasma. From the resonance curves[9] we know that for our intensity the plasma must be tuned to the correct density to within 6%. Interferometric measurement of the θ–pinch plasma showed that at the peak of compression plasma with the resonant density of 5.8×10^{16} cm^{-3} with a density scalelength of 20 cm could indeed be formed in a variety of gases at different fill pressures. Unfortunately, we discovered that at the peak of compression up to a few kG of the magnetic field remained trapped inside the plasma column.

Thus although we could excite a large plasma wave the electron injection proved to be impossible at the injection energy of 1.5 MeV. We therefore tried two other conditions where resonant density could be obtained. These will be discussed later in the paper. (3) The electron linac is an 9.3 GHz, x band travelling wave structure with a thermionic gun. The output energy is 1.5 MeV and the average current is about 2 mA at the laser focus. The macropulse is 3 μs long and contains 20 ps micropulses each separated from the next by ~110 ps. Each micropulse contains a few million electrons focused to a spot size of $2\sigma_\perp \cong$ 1 mm. This large spot size is a result of the large emittance introduced when the beam scatters in a 6 μm window at the end of the linac. (4) The detection apparatus consists of a grating spectrograph with either a pyroelectric array or an optical streak camera as the detection element.

4.2. Results to-date

Fig. 6 shows the sequence of events during a typical experimental shot. The theta-pinch is fired first. We overfill the plasma chamber with He such that at the peak of the compression the plasma density is greater than the resonant density As time progresses, the external magnetic field goes through zero again (at t = 7.5 μs in Fig. 6). By this time the plasma density drops to the resonant value. Although the plasma is not as uniform as it is at the peak of the compression it still has a fairly large density scalelength.

Another condition for the operation of the theta-pinch was to use a very high fill pressure (1.75 Torr) and use the electric field of the theta-pinch as a source of pre-ionization. In this case, heating from the laser beam contributes significantly to the plasma density. Indeed, Raman backscatter measurements indicated that the plasma was rather inhomogeneous with density variations of up to 50% from the resonant value.[11]

An estimate of the amplitude of the plasma wave, a_w, can be obtained by measuring the power in one of the forward scattered electromagnetic sidebands to the laser frequency. The generation of Stokes (frequency downshifted) and anti-Stokes (frequency upshifted) sidebands at frequency shifts $\pm\omega_p$ is necessarily part of the beatwave excitation process. One can think, for example, of the first Stokes sideband as arising when the lower frequency pump Thomson scatters off the plasma wave. Thus the Stokes level is a function of the amplitude-length product of the plasma wave. Using this fact we estimate[11]

$$(a_w L)\ \text{experiment} \leq 0.13\ \text{mm} .$$

From Raman scattering threshold we estimate L, the interaction length to be between 1 and 2 mm which means that $a_w \cong 0.065$ to 0.13 and $N \cong 6$ to 12. Clearly, although we were obtaining reasonable strengths for the undulator, the number of undulator periods was limited by the poor plasma inhomogeneity.

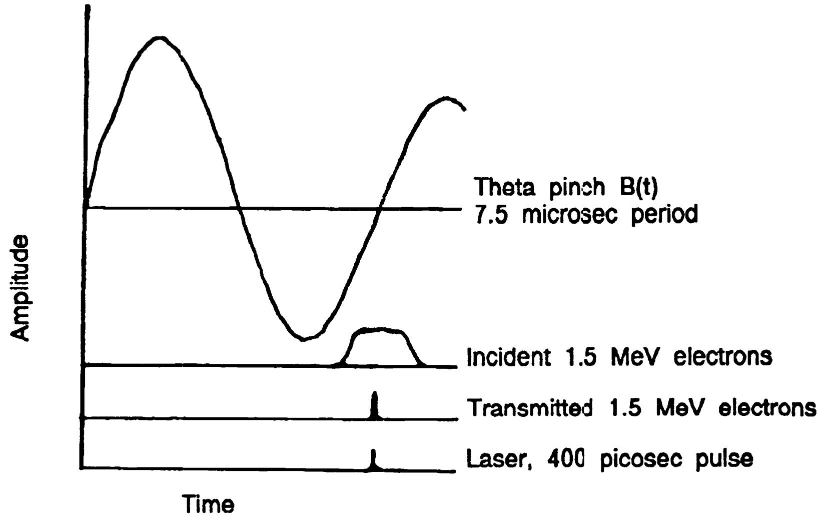

Figure 6. The relative timing of the theta pinch, the linac pulse, the transmitted linac pulse, and the CO_2 laser pulse.

4.3. Electron injection into the beat wave

We nevertheless tried to inject the electrons into the beat wave. One reason for this was to find out whether at B = 0 the plasma was truly field free or not. Using a 1:1 imaging lens the electron beam emerging from the plasma was imaged on a fluorescent screen. Without any plasma we find that electrons within a narrow time window of 50 ns about the B = 0 emerge from the focal region undeflected. But with the plasma even these electrons are radially deflected randomly due to the trapped field. From the radial displacement of the image[12] we have been able to estimate the magnitude of the localized trapped magnetic field inside the plasma when B = 0. We find that typical magnitude of the trapped field is several hundred Gauss. As a consequence of this trapped field, the injected electrons can no longer deterministically be made to overlap with the undulator. Consequently, we are currently developing a new plasma source for generating the plasma undulator.

4.4. Development of a new plasma source

As discussed earlier, electron injection into the plasma wave was hampered by the lack of a suitably magnetic-field-free plasma source at the desired high electron number density (6×10^{16} cm^{-3}). The present experiment relaxes the density requirement to 8×10^{15} cm^{-3} by changing the two laser wavelengths. Accordingly, we now operate the CO_2 laser on the 10.6 and 10.3 μm wavelengths rather than the 10.3 and 9.6 mm wavelengths. We have replaced the theta pinch with a multi-cathode arc plasma source. The electrodes consist of a 3 cm square metal anode and a 3 × 1 cm plastic and metal cathode as shown schematically in Fig. 7.

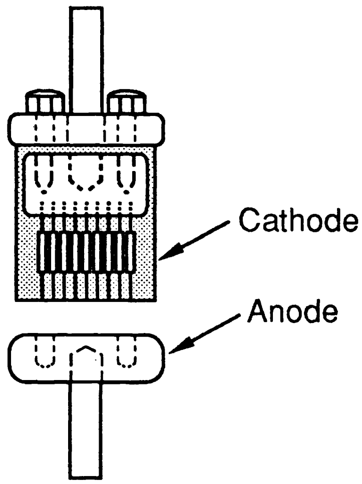

Figure 7. Schematic of arc-plasma electrode assembly.

The cathode is really a 7×2 array of 1.3 ohm, 1/2 W carbon resistors embedded in an insulating casting resin which has been machined off to expose the wire leads. This idea is from the early days of TEA CO_2 lasers.[13] The purpose of the elaborate cathode is to prevent the discharge current from flowing from a small spot which would severely limit the length of plasma. The circuit, operating at 10 μF and 2 kV, is over-damped with a one ohm resistor. The peak current is only around 2 kA so that the magnetic field at, say, 1 cm from the center of the plasma is a manageable 400 Gauss. We have tested this and similar cathodes and find, photographically, that the plasma is quite uniform over the 1 cm length of the resistor array and that the peak density can be adjusted to the desired resonant density. We are currently beginning our experiments again on the excitation of the undulator using two laser frequencies.

5. CONCLUSIONS

We have described the status of the experimental and theoretical work on the generation of spontaneous emission using a plasma wave undulator at UCLA. The first phase of experiments has been completed and has yielded plasma waves with $a_w \sim 0.1$ and $N \lesssim 10$. Electron injection in such waves has been hindered by trapped B fields inside the plasma column. A new plasma source has been devised and is ready for testing. Detailed theoretical and numerical work in support of the experimental program is also being carried out.

6. ACKNOWLEDGMENTS

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