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Comprehensive Studies of Oxygen Transfer under Nonideal Conditions

A dissertation submitted in partial satisfaction of the
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in Engineering

by

Hyung Joo Hwang

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The dissertation of Hyung Joo Hwang is approved.

Jane L. Valentine

Jane L. Valentine

Jeffrey I. Zink

John A Dracup

John A. Dracup

William W-G. Yeh

Michael K Stenstrom

Michael K. Stenstrom, Chairman

University of California, Los Angeles

1983

To my parents

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LIST OF SYMBOLS

A = interfacial area

a = surface to volume ratio

a_C = proportionality constant, defined by equation 54

B = number of impeller blades

b_C = constant value of K_La, defined by equation 55

C = effective average dissolved oxygen concentration in the liquid phase

C_i = influent dissolved oxygen concentration

C_o = initial dissolved oxygen concentration

C_S = saturation dissolved oxygen concentration

C_∞^* = saturation dissolved oxygen concentration attained at infinite time

C_R = defined by equation 42

C_R' = defined by equation 61

C = constants (with alphanumeric subscripts)

C_D = drag coefficient

D = liquid depth

DI = impeller diameter

D_L = oxygen molecular diffusion coefficient in water

d_B = equivalent bubble diameter

d_{vs} = ratio of bubble volume to surface area (sauter diameter)

f = mass transfer during time of exposure per unit surface area

f = correction factor for air flow rates (with subscripts)

Fr = Froude number, defined by equation 27

G = mass transfer rate

G_S = gas flow rate

G = constant (with subscripts of 1 throuth 4, defined by equations 80 through 83)

g = gravitational constant

HI = impeller blade height

HP = horse-power input

h = liquid height

Kk = exponent defined in equation 44

K_L = liquid film coefficient

K_La = volumetric mass transfer coefficient

K_La' = defined by equation 60

Ku = first order reaction constant for substrate removal

LI = impeller blade length

L(t) = substrate concentration

m = mass of oxygen

n = impeller rotational speed

Ne = Newton number, defined by equation 28

P = power input

p = pressure

Q = mixed-liquor flow rate

R = mass transfer rate per unit time and unit interfacial area

R_c = constant oxygen uptake rate, defined in equation 47

Re = Reynolds number, defined by equation 26

r = equivalent bubble radius

R(t) = microbial oxygen uptake rate

R_0 = proportional constant, defined in equation 47

r_s = average surface renewal rate

S = impeller Pitch

S_B = objective function for Box method

Sc = Schmidt number, defined by equation 28

S_L = objective function for linearized model

$S(t)$= mixed-liquor suspended solid concentration

T = temperature

u = bubble terminal velocity

V = liquid Volume

v_B = bubble rising velocity

W = aeration tank width

We = Weber number, defined by equation 29

WI = impeller blade width

Y = transfer number, defined by equation 34

Y_H = transfer number, defined by equation 67

<u>Subscripts</u>:

B = bubble

comp= compressed air

m = model

p = prototype

std = standard condition

TW = tap water

WW = wastewater

v = vapor

<u>Greeks</u>:

α = alpha factor, defined by equation 20

β = beta factor, defined by equation 21

θ = theta factor, defined by equation 22

ϵ = bubble hold up ratio

ν = kinematic viscosity

ρ = liquid density

σ = surface tension

ϕ = time of exposure

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VITA

May 12, 1949 ----- Born, Seoul, Korea

1972 ----- B.S. in Engineering, Seoul National University, Seoul, Korea

1979 ----- M.S. in Engineering, University of California, Los Angeles

1977-1979 ----- Research Assistant, School of Engineering, UCLA

1979-1981 ----- Teaching Assistant, School of Engineering, UCLA

1981-1983 ----- Post Graduate Research Engineer, School of Engineering, UCLA

PUBLICATIONS

Stenstrom, M.K., L.C. Brown, and H.J. Hwang, "Oxygen Transfer Parameter Estimation," J. Env. Eng. Div., Am. Soc. Civ. Engrs., Vol.107, No.EE2, pp.379-397, 1981.

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ABSTRACT OF THE DISSERTATION

Comprehensive Studies of Oxygen Transfer under Nonideal Conditions

by

Hyung J. Hwang

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Professor Michael K. Stenstrom, Chairman

This research is concerned with oxygen transfer mechanisms in the wastewater treatment process. Diffused and surface mechanical aeration are evaluated in these studies.

For diffused aeration studies, mathematical models are developed for oxygen transfer in activated sludge mixed-liquor for various conditions of microbial oxygen uptake rate, and the models are applied in a pilot scale aeration column. Experiments are performed in tap water, with and without a synthetic surfactant, and mixed liquor. Liquid depths are tested in the range from 5 to 15 ft. The experimental results show that the volumetric mass transfer coefficient linearly increases with air flow rate in all liquids. In the solution of tap water and synthetic surfactant, the alpha factor varies in the range of 0.4 and 0.8, and its value is dependent on the other contaminants as well as the surfactant concentration. In mixed-liquor tests, the volumetric mass transfer coefficient is linealy dependent on the microbial oxygen uptake rate, which is functionally related to the degree of wastewater treatment, for the given process conditions. The alpha value in mixed liquor varies in the range of 0.4 and 0.9, and its value increases with increasing degree of wastewater treatment.

The alpha factor is affected by air flow rate and liquid depth. The alpha factor decreases with increasing air flow rate and liqid depth above diffuser. Column test results are compared with results in full scale aeration facilities.

For surface aeration, nondimensional parameters are derived using similarity theory, and the relationships between them are established from experimental data using three laboratory-scale aeration tanks. Experiments are performed in tap water and tap water containing surfactants, over a concentration range 0 to 15 mg/l of dodecyl sodium sulfate, an anionic detergent. Oxygen mass transfer rate is dependent to the -0.8 power of Weber number and 2.22 power of Froude number.

I. INTRODUCTION

The activated sludge process is currently the most popular method of biological wastewater treatment and its popularity is increasing. Reliable operation of this process is dependent upon an aeration system to supply dissolved oxygen to the mixed-liquor. Aeration is now the most energy intensive aspect of wastewater treatment and consumes as much as sixty to eighty per cent of the total energy requirements in a modern wastewater treatment plant (Houck and Boon, 1981). The current trend is to replace older, lower-efficiency aeration systems, such as coarse bubble spiral roll systems, with more energy efficient systems, such as fine bubble diffusers with a full-floor coverage (Aberley et al.,1974; Bacon et al., 1977, Houck and Boon,1981; Stenstrom et al., 1983).

Unfortunately, the phenomena underlying aeration process have not been fully understood, and a universal basis for design and operation does not exist. One of the most urgent needs is the development of mathematical models and proper experimental techniques to predict and evaluate the performance of aeration systems under process or nonideal conditions. Furthermore, oxygen transfer rate from gas to liquid phase is dependent on such factors as method of aeration, power input intensity, mixing intensity or turbulence, temperature, test facility geometry, physicochemical properties of the liquid, and other unknown factors. Even though the designer or operator can fix or control some of these parameters, successful design requires the knowledge of the effects of uncontrollable parameters, especially the liquid properties, through field measurable quantities, such as microbial oxygen uptake rate or surface tension. Microbial oxygen uptake rate is a good indication of level of wastewater treatment.

Aeration devices are conventionally evaluated in clean water, and the results are extrapolated to the actual operating conditions through the widely accepted parameters; alpha, beta, and theta, and nondimensional scale-up methods. The inherent problem in this approach is the dependency of alpha and beta on the actual operating conditions, and the characteristics of the air-liquid interface, which have not been well defined by similarity theory. Especially, alpha factor changes substantially under the presence of trace contaminants. An alternate approach used by the British which partially overcomes this drawback is to simulated the contaminants in wastewater by adding a specified amount of surface active agents (SAA or surfactants) to tap water. An extensive study of the surfactants, scale-up methods, and the parameters relating the test results in clean water, water containing surfactants, and mixed-liquor are required for better understanding of the performance of aeration systems.

The three basic categories of the aeration methods are:

(1). Surface or mechanical aeration methods, which increase interfacial area by spraying water droplets into the air,

(2). Diffused aeration methods, which release air bubbles beneath the surface of the water,and

(3). Combined or turbine aeration methods, which introduce large air bubbles into water and reduce their sizes mechanically.

In this research, two aeration methods, surface and diffused aeration, have been studied. For the surface aeration method, a group of nondimensional numbers were formulated using similarity theory, and the relations between them were experimentaly established. For the diffused aeration method, mathematical models were developed for nonsteady-state aeration of mixed-liquor at various conditions of microbial oxygen uptake rate, and the volumetric mass transfer coefficients were evaluated. The mass transfer rates under different liquid conditions were analyzed.

II. LITERATURE REVIEW

A. GAS TRANSFER THEORIES

A-1. TWO FILM THEORY

Among the theories describing gas transfer through the gas-liquid interface, the best known and most widely used theory is the "two film theory", which was first established by Lewis and Whitman (1924). The basic concept of this theory is the existence of two thin films on each side of the interface. For gas molecules to dissolve into the liquid, they must pass through those two films by a slow molecular diffusion process. The concentration of dissolved gases at the liquid film interface is in equilibrium with the gas phase as defined by Henry's Law. The concentration in the bulk liquid beneath this film is maintained uniform at all points by turbulent mixing. The liquid film is free from turbulence and the transfer across the films is at steady state conditions. The resistances to transfer of solutes at the gas and liquid films are considered in series.

In case of a sparingly soluble gas in a liquid, e.g., oxygen in water, transfer rate is controlled by the resistance in liquid film, and can be expressed as:

$$\frac{dm}{dt} = K_L \cdot A \cdot (C_S - C) \qquad (1)$$

where m = mass of oxygen

K_L = liquid film coefficient

A = interfacial area normal to mass transfer

C_S = saturation dissolved oxygen concentration

C = dissolved oxygen concentration in the liquid plase

Equation 1 divided by liquid volume gives concentration change with respect to time as follows:

$$\frac{dC}{dt} = K_L \cdot \frac{A}{V} \cdot (C_S - C) \tag{2}$$

$$= K_L a \cdot (C_S - C) \tag{3}$$

where $K_L a$ = volumetric mass transfer coefficient

V = liquid volume

a = specific interfacial area (interfacial area per unit volume)

The principal limitation of the two film theory is in the assumption of a steady-state condition in the stagnant film. Other theories have been developed to overcome this limiting assumption.

A-2. PENETRATION THEORY

The steady-state assumption in a liquid film in the two film theory is not required in penetration theory. Higbie (1935) studied the liquid film resistance of sparingly soluble gas in water. His concept was that when the gas and liquid are brought into contact, the concentration of solute in liquid film is the same as that of the bulk liquid. At first, the gas molecules penetrate the gas-liquid interface and accumulate there to create a high concentration gradient at the liquid layer beneath the interface. This high initial concentration gradient produces fast diffusion rate in the liquid layer and it decreases with time to reach a linear steady state gradient. Gas transfer during this penetration period is significant when the period of contact between gas and liquid is not much longer than the penetration period.

The mathematical presentation of molecular diffusion through liquid layer is given by Fick's law as follows:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} \tag{4}$$

where D_L = diffusion coefficient

The initial and boundary conditions are:

$C = C_0$ at $t = 0$, $x \geq 0$

$C = C_S$ at $t > 0$, $x = 0$

$C = C$ at $t > 0$, $x = \infty$

The equation 4 can be solved and the gas transfer during the time of exposure per unit interfacial area is:

$$f = 2\sqrt{\frac{D_L \phi}{\P}} \; (C_S - C) \qquad (6)$$

The gas transfer rate per unit time and per unit area is obtained by dividing equation 6 with time of exposure.

$$R = \frac{f}{\phi} = 2\sqrt{\frac{D_L}{\P \, \phi}} \; (C_S - C) \qquad (7)$$

By comparing equations 3 and 7, the liquid film coefficient can be obtained.

$$K_L = 2\sqrt{\frac{D_L}{\P \, \phi}} \qquad (8)$$

where f = mass transfer during time of exposure per unit surfacial area

R = mass transfer rate per unit time and unit interfacial area

ϕ = time of exposure

A-3. SURFACE RENEWAL THEORY

Penetration theory is based on a concept requiring constant exposure time of each stagnant film. Danckwerts questioned the assumption of a stagnant film of liquid at the interface and suggested surface renewal theory (1951). He assumed that the turbulence of liquid extended to the surface and replaced with fresh surface those older parts that had been exposed for a finite length of time in a random manner. He derived the following expression:

$$K_L = \sqrt{D_L \cdot r_s} \tag{9}$$

where r_s = average surface renewal rate (a rate of production of fresh surface per unit total surface available)

Many theories have been proposed in addition to those previously mentioned (Dobbins, 1955; Whitaker and Pigford, 1966; Angelo and Lightfoot, 1968; Springer and Pigford, 1970; Danckwerts, 1970; Nguyen et al., 1979), but an important point is that equation 3, with K_La as one parameter, is applicable in all theories.

The major difficulty in applying these theories, other than two film theory, is the need to independently determine the liquid film coefficient and interfacial area. The determination of the interfacial area requires measurements of bubble or droplet size and their life time in water or air, and the statistical knowledge of their distribution. Hence most experimental works have been done in bench scale facilities using photographic techniques, and their results are usually not applicable to aeration studies. Liquid film coefficient determination has a more serious problem. No appropriate hydrodynamic theory to correlate surface film thickness or surface renewal rate to turbulence, with any operating parameters, has been developed. Due to the difficulty of measuring interfacial area under process conditions, most investigators have measured $K_L a$ as opposed to K_L. If it is desirable to know K_L, it is usually determined from $K_L a$ and the estemates of interfacial area. Therefore, for better understanding of the variation of $K_L a$, it is important to review the available literatures on K_L and a separately, as well as $K_L a$.

B. LIQUID FILM COEFFICIENT

B-1. EFFECT OF BUBBLE SIZE

Higbie's penetration model (1935) implied that the liquid film coefficient K_L was proportional to one half power of diffusion coefficient of gas molecules in liquid phase. This exponent of 0.5 was questioned by many investigators and others reported values from 0.5 to 1.0.

Calderbank (1959) showed that the exponent was dependent on bubble size and reported values of 0.86 and 0.67 for large and small bubbles, respectively, in an agitated gas-liquid contact tank. The value of 0.67 for the small bubbles was also applicable to various gases in aqueous solution of surface active materials. He reported that mixing intensity, bubble size, gas hold-up and mean residence time of gas bubbles in aerated mixing vessel with sieve plate had no appreciable effect on K_L over the range of 0.2 to 0.5 cm of bubble diameter which was in the regime of a "large bubble". But Calderbank stated in the same report, that the value of the exponent for the aqueous solution of "different solutes" was 0.5 while keeping viscosity constant, and left some question on the role of viscosity for the deviation.

Alternatively, Ippen and Carver (1954), Camp (1958), and Akita and Yoshida(1974) confirmed the value of 0.5, and O'Connor and Dobbins (1958) reported the value of 1.0 for exponent.

As stated earlier, Calderbank (1959) reported bubble size had no appreciable effect on K_L in the small bubble or large bubble region. Calderbank and Mooyoung (1961) confirmed this finding and they stated the small bubbles behaved as solid spheres. Small bubbles (dia. < 0.2 cm) and large bubbles (dia. > 0.25 cm) had constant K_L values of 0.042 and 0.135 mm/sec (15 and 50 cm/hr), respectively, in water and aqueous solutions. In the transition region, K_L decreased as bubble diameter was reduced, and its drop was "sharp and critical".

The dependence of K_L on bubble diameter has been reported in diffused aeration systems. Eckenfelder (1959) showed that oxygen transfer from air bubbles rising through the water could be correlated to the Sherwood number, the bubble Reynolds number and liquid depth. The correlation can be reduced to:

$$K_La = C_a \cdot \frac{v_B}{h^{1/3}} \tag{10}$$

where C_a = constant
v_B = bubble rising velocity
h = liquid depth

In this expression, the dependency of K_L on bubble velocity implies the dependency of K_L on a bubble diameter.

Barnhart (1969) reported K_L values over the large range of bubble size. According to Barnhart, K_L values increased rapidly from about 35 cm/hr to maximum value of 210 cm/hr as bubble size grew from 0.05 cm to 0.22 cm diameter in diffused aeration. When bubble diameter increased beyond this point, K_L decreased rather sharply. Carver (1969) presented data showing K_L increased from 10 to 110 cm/hr as bubble size grew from 0.025 to 0.1 cm.

B-2. SURFACE RENEWAL RATE

In the surface renewal model, mass transfer coefficient is proportional to 0.5 power of surface renewal rate. The surface renewal rate in gas-liquid dispersion system is dependent on the bubble diameter and the slip velocity of bubble relative to the surrounding liquid. Garner and Hammerton (1954) found that for bubbles with diameter over the range of 0.3 to 0.8 cm, the time for surface renewal was the same as the time for the bubble to travel one bubble diameter in water column.

Camp (1958) developed a similar relationship from the data of Ippen and Carver (1954), as follows:

$$K_L = C_b \cdot \sqrt{\frac{D_L \cdot v_B}{d_B}} \qquad (11)$$

where C_b = experimental constant

d_B = equivalent bubble diameter

O'Connor and Dobbins (1958) showed that surface renewal rate approximately equalled to the velocity gradient in the liquid at the interface.

Eckenfelder (1959) found that K_L was inversely proportional to one-third power of water depth as shown in equation 10. He explained this due to higher K_L value during bubble formation and burst on the surface. This was observed also by Downing and Boon (1963).

The liquid film coefficient for rising bubble varies over the surface of the bubble. Adeney and Becker observed that air bubbles of 9 cm/sec exhibited K_L value of 230 cm/hr at the tip where the higher surface renewal rate was expected due to surface area expansion, while 32 cm/hr at the side of the same bubble (cited by Barnhart, 1969). Akita and Yoshida (1974) averaged K_L values over the surface and derived following expression for the relation of K_L and K_La;

$$K_L = \frac{K_La}{6\,\epsilon} \cdot d_{vs} \qquad (12)$$

where d_{vs} = bubble volume-surface area diameter

ϵ = gas bubble holdup (ratio of trapped gas volume to water volume)

After nondimensional analysis, they derived

$$K_L = 0.5 \cdot g^{5/8} \cdot D_L{}^{1/2} \cdot \rho^{3/8} \cdot \sigma^{-3/8} \cdot d_{vs}{}^{1/2} \qquad (13)$$

where g = gravitational constant

ρ = liquid density

σ = surface tension

and explained the effect of surface tension on mass transfer coefficient as its effect on bubble shape. Their finding on the role of surface tension did not agree with the report by Calderbank and Mooyoung (1961) who stated the K_L value of a large bubble in the aqueous solution of surface active material had lower value than a small bubble or a rigid sphere. Bilstad and Lightfoot (1980) in a research report on U-tube aeration stated the ratio of K_L values of oxygen and nitrogen was 0.86 which was the same value as the square root of diffusion coefficient of those gases in water.

C. SURFACE TO VOLUME RATIO

C-1. BUBBLE SIZE

Surface to volume ratio primarily depends on gas bubble or liquid droplet diameter. For a given air flow rate, the increase of bubble diameter causes the reduction of the bubble surface area. For example, a 0.4 cm diameter bubble has 50% less total surface area compared to 0.2 cm diameter bubble for a given gas flow rate assuming an identical terminal velocity. Considering the increase of bubble velocity caused by the larger diameter, the surface to volume ratio is reduced further.

Bubbles in deep water body undergo two phases; formation and rise. Akita and Yoshida (1974) reported the initial bubble size released from a submerged orifice was independent of the properties of the systems, such as surface tension, viscosity, density of gas and liquid, but dependent only on orifice diameter and gas velocity in the orifice, while the size of bubbles rising through 2.5 m water column was dependent primarily on a balance between the coalescence and break-up rates. They also reported that the distribution of bubble size produced from a single orifice approximately followed the logarithmic normal distribution law, in the range of orifice diameter of 0.04 to 0.8 cm. Urza and Jackson (1975) observed similar results in a column of 3-inch diameter and 55-foot length and reported that bubbles produced from 0.25" orifice were slightly larger than those from 0.125" orifice, but after rising to 75% of the height, the sizes became similar.

Akita and Yoshida's observation (1974) of no influence of surface tension on initial bubble size can be explained with the concept of dynamic surface tension. In aqueous solution of surface active agents, the freshly formed surface exhibits surface tension close to the value of pure water. As the surface active materials migrate to the surface and build up an excess surface concentration, the dynamic surface tension decreases until it reaches the final value of static surface tension. At a high gas flow rate per orifice, the high frequency of bubble formation does not allow enough time for any significant decrease of dynamic surface tension and consequently the initial bubble size remains essentially the same in aqueous solution of surface active materials as in pure water.

C-2. BUBBLE SHAPE

Bubble shape is an another important factor affecting gas transfer in the diffused aeration. Haberman and Morton (1956) investigated extensively the shape and motion of air bubbles in various liquids and observed that as the bubble size increased, a change in bubble shape from spherical to ellipsoidal and from ellipsoidal to spherical cap occured in all liquids.

For very small bubbles, surface tension dominated and made the surface area of the bubble as small as possible, i.e. that of a sphere. The spherical shape was maintained up to bubble Reynolds number of 400. As the bubble size grew, the surface force became less important in comparison to the viscous and hydrodynamic forces, and flattening of the bubble occurred to produce an oblate ellipsoidal shape. As the bubble size grew further, the viscous and surface tension forces became much smaller relative to the hydrodynamic forces, and the bubbles had the shape of spherical caps. The upper surface of a spherical cap was essentially spherical while the lower surface varied from highly irregular for low viscosity liquids to smooth for very high viscosity liquids.

For a bubble rising at its terminal velocity, the drag coefficient can be defined as:

$$C_D = \frac{(8/3) \cdot g \cdot r_e}{u^2} \tag{14}$$

where C_D = drag coefficient
g = gravitational constant
r_e = equivalent radius of bubble
u = terminal velocity of bubble

and Haberman and Morton (1954), obtained a constant drag coefficient of 2.6 for spherical bubbles which could be rearranged to give an expression for terminal velocity of bubbles:

$$u = 1.02\sqrt{g \cdot r_e} \tag{15}$$

For spherical bubbles, from the results presented by Haberman and Morton (1956), terminal velocity varied with (4/3) power of equivalent radius.

Ippen and Carver (1954) observed that bubbles in size between 0.2 and 0.6 cm in diameter in water had approximately the same terminal velocity of about 23 cm/sec which was the value obtained by Haberman and Morton.

C-3. BUBBLE PROXIMITY

Reviews aforementioned are restricted to single bubble observations without any proximity effect from adjacent bubbles. Garner and Hammerton (1954) noticed the proximity effect when the bubbles were separated by the distance to travel as much as 3 seconds. They found that rising velocity was appreciably higher when the bubble was affected by adjacent bubbles. The data reported by Pasveer (1955), Barnhart (1969) and Baars (1955) showed about 20 to 30 % increase of velocity of swarm bubbles compared to single bubbles. Figure 1 summarizes their data and the data of others. In large aeration tanks, as opposed to small columns, the different effect of proximity can be observed. Ippen et al. (1954) mentioned that in full size aeration tank, the terminal velocities were slightly lower due to interference, increased turbulence and eddy currents from adjacent bubble.

Symbol	Bubble	Liquid	Reference
——	Single	Tap Water	Haberman and Morton, 1956
-----	"	Distilled or Filtered Water	" "
△	"	Aqueous Solution of Glim	" "
○	"	Contaminated with Vaseline	Garner and Hammerton, 1954
×	Swarm	Tap Water	Pasveer, 1955
+	"	SAA Solution (0-53.7 mg/l)	Barnhart, 1969

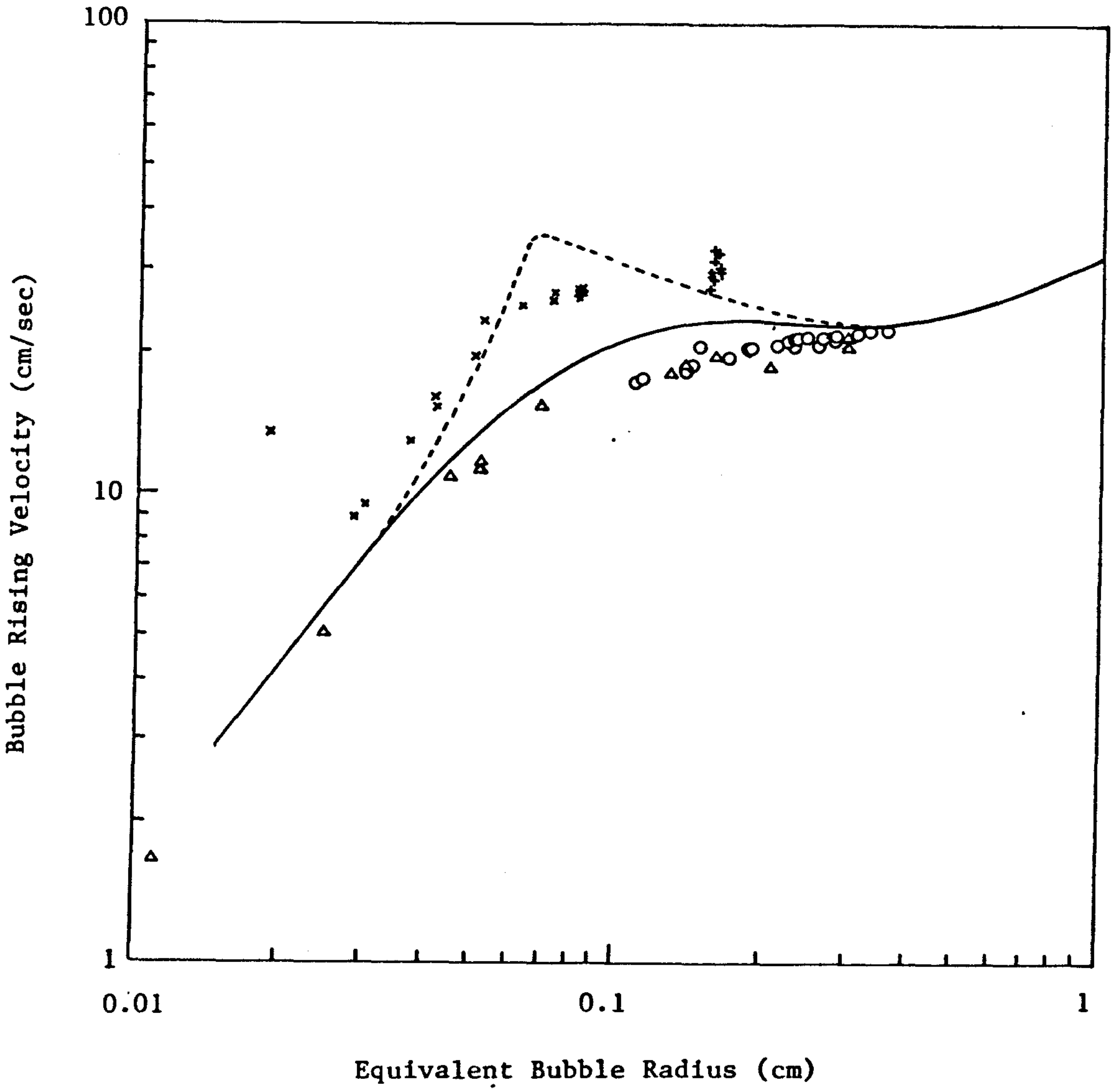


Figure 1. Effect of Proximity on Bubble Velocity

C-4. BUBBLE MOTION

As the bubble shape changes with increasing bubble size, bubble motion also changes. Spherical bubbles have either rectilinear or helical motion, and ellipsoidal and spherical cap bubbles have both motions as well as rectilinear motion with rocking. These motions occur with bubble Reynolds number as follows;

Rectilinear motion .... for $Re < 300$
Helical motion .... for $300 < Re < 3000$
Rectilinear with rocking ... for $Re > 3000$

The transitional Reynolds numbers corresponded to bubble diameter of 0.1 and 1.2 cm, respectively, in distilled or filtered water. Garner and Hammerton (1954) observed the same phenomena of bubble shape and motion from straight to helical at bubble diameter of 0.1 cm in water. In the helical or zig-zag motion, the pitch and amplitude were both approximately two bubble diameters. The size of the helix increased with bubble diameter up to 0.5 cm, when the straight vertical rise interupted the helical path. When bubble diameter was greater than 0.8 cm, bubbles nearly always rose in straight lines.

The critical diameters observed by different investigators did not always agree. The transition from straight to spiral motion was reported at 0.15 cm by van Krevelen (1950) and 0.14 cm by Saffman (1956), and the transition from spiral to rectilinear with rocking motion at 0.6 cm in two later reports. Saffman (1956) disagreed with Garner and Morton on the helical motion. He stated the radius of spiral was about 0.15 cm when bubble diameter was 0.2 cm, but it decreased as bubble diameter increased and amplitude of zig-zag was also in this order. The frequency of zig-zag was approximately 7 per second, independent of bubble size. The angular velocity of spiral was about 30 radian/second (5 RPS) and also independent of bubble size.

C-5. EFFECT OF SURFACE ACTIVE AGENTS

Chemically, there are two classes of surface active agents, the ionics and the nonionics. The nonionic surface active agents have non-ionizable hydrophilic end groups which contain a number of oxygen, nitrogen or sulfur ions. The ionic surface active agents have two divisions: cationics, in which an active portion or hydrophilic end is a cation, and anionics in which an active portion is an anion. Anionics make up 73 % of the U.S. consumption of surfactants (Rosen, 1978).

Surface Active Agents (SAA) have been shown to have great effects on bubble shape and motion. Even though the surface tension may not be reduced at the initial stage of bubbles or liquid droplets, it becomes important as the surface ages.

Haberman and Morton (1956) reported signifcant difference of bubble velocity in distilled and tap water. Filtered tap water gave same high velocity as distilled water. This effect occurred mainly in the region of spiral path of bubble diameter between 0.07 and 0.5 cm with maximum difference of velocity, about 20 cm/sec at diameter of 0.13 cm. The presence of minute particles in ordinary tap water, most of which could be removed by filtering, apparently caused these observed changes.

The surface active materials in water further reduce bubble velocity. Garner and Hammerton (1954) noted the reduced velocity of bubbles whose surfaces were contaminated with vaseline. Haberman and Morton (1956) presented results of bubble velocity in 0.42 % by volume of a liquid detergent, glim. The aqueous glim solution did not alter viscosity and density of water but reduced surface tension by 40 dynes/cm. Again the velocity deviation from that of tap water was confined in the region of helical motion.

Garner and Hammerton (1954) also noticed that the contaminant changed bubble motion from helical to zig-zag. The changes of bubble shape and size also resulted when the water was contaminated with SAA. When clean bubbles rise through a fluid containing dissolved surface active agents, the surfactant molecules are adsorbed on the bubble surface. The molecules are convected to the rear of the bubble and accumulated there to form a "cap". A surface tension gradient which opposes the tangential shear stress stops the surface flow over the rear portion of the surface. This cap will grow with the concentration of SAA until the maximum size that the surface tension grandient can support (Davis and Acrivos, 1966). The effect of the cap is most significant when the bubble motion is most observable. A small bubble behaves as a solid sphere by itself so that the cap does not have a significant effect.

Most researchers reported that the bubbles became more spherical as surface active agent concentration increased (Baars, 1955; Akita and Yoshida, 1974). Carver (1956) reported that the presence of detergent reduced bubble size from venturi tube, but exhibited no effect on the bubble from capillary tubes. However, McKeown and Okun (1953) observed opposite results, in which bubbles became less spherical as SAA concentration increased and the ratio of bubble width (major axis) to equivalent diameter increased with increasing alkyl benzene sulfonate (ABS) concentration, to a maximum value at the critical micelle concentration, then decreased slightly at higher concentration. Harmathy (1960) presented experimental results showing the ratio of major to minor axis increases with Eotvos number meaning that, for the same bubble size, the bubble in detergent solution ($\sigma < \sigma_{TW}$) became less spherical.

Internal circulation of bubbles or liquid droplets is an another important factor for molecular diffusion of gas in the gas-liquid interface. Bubbles with diameter less than 0.015 cm rose as rigid spheres, which implies that there is no internal circulation of gas phase in bubble. K_L value of 45 cm/hr at bubble diameter of 0.03 cm and 170 cm/hr in fluid sphere indicated circulation started when the bubble diameter was between 0.015 cm and 0.03 cm, but it did not reach maximum speed until the bubble grew to 0.25 cm diameter (Haberman and Morton, 1956) in tap water. Circulation at bubble diameters up to at least 0.6 cm could be stopped with very small quantites of surface active materials absorbed on the gas-liquid interface (Garner and Hammerton, 1954). Absorption of surface active materials on the interface also made air bubbles more rigid (Baars, 1955; Calderbank and Mooyoung, 1961; Mancy and Barlage, 1968).

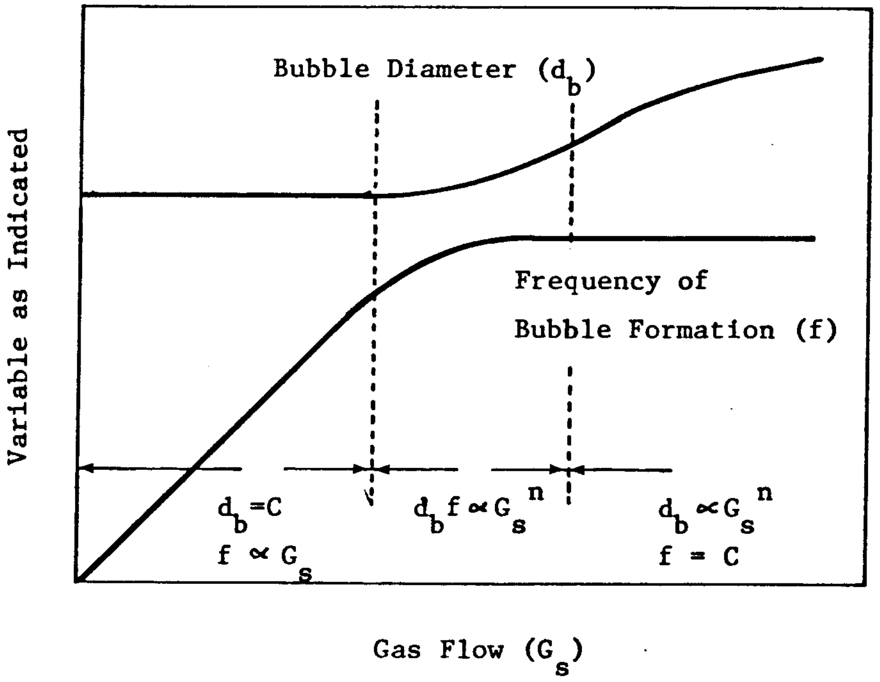


Figure 2. Bubble Diameter and Frequency vs. Gas Flow Rate

(After McKeown and Okun, 1953)

C-6. GAS FLOW RATE

One more variable to be considered for bubble size is gas flow rate. For the large orifices, e.g., a sparger, excessively large bubbles are formed initially, but the large bubbles are broken down by the hydrodynamic forces, so that the gas flow rate does not have a significant role. However, for a small opening diffuser, gas flow rate does have effects on bubble size determination. At a low gas flow rate per opening, if enough time is allowed for a bubble to grow individually, without any interuption by the following gas stream, bubble size will remain constant, and only frequency of formation increases as gas velocity increases. At a high gas flow rate, before the fully grown bubble is detachted from the orifice, the following gas is added, and bubble size increases. Eckenfeder (1959) proposed that for the range of gas flow rates typically encountered in working porous plate diffusers, the bubble size was an exponential function of the gas flow rate. McKeown and Okun (1953) presented figures showing the qualitative relations of bubble formation frequency, diameter, and gas flow rate as shown in Figure 2.

Additional details can be found in the literature, for spiralling bubbles (Saffman, 1956; Harmathy, 1960), for spherical cap bubbles (Parlange, 1969; Wegner, 1973), and the effect of surfactants (Garner and Hammerton, 1954; Baars, 1955; Pasveer, 1955; Saffman, 1956; Zieminski et al., 1967; Mancy and Barlage, 19681 Tsuge et al., 1981).

D. VOLUMETRIC MASS TRANSFER COEFFICIENT

D-1. AIR FLOW RATE

Except for a few observations, the volumetric mass transfer coefficient (K_La) increases almost linearly with air flow rates (McKeown and Okun, 1953; Urza and Jackson, 1975; Bacon et al., 1977; Gilbert and Libby, 1979; Hwang and Stenstrom, 1979; Tewari and Bewtra, 1982). This means that with increasing air flow rate, bubble diameter increases, so that the surface to volume ratio (a) decreases per unit air flow rate, but that the liquid film coefficient (K_L) increases to compensate.

Jackson and Shen (1987) and Jackson and Hoech (1977) related K_La value to the power of superficial air velocity, and found that the exponent varied from 1.08 to 1.13. Eckenfelder (1959a) derived a nondimensional expression for K_La values which can be reduced as follows:

$$K_La = \frac{6 \cdot C_c \cdot G_s \cdot h^{2/3}}{d_B \cdot V} \qquad (16)$$

where G_s = gas flow rate

C_c = constant

Since bubble diameter varies with gas flow rate over the range used in practice, the relation can be simplified to:

$$K_La \sim G_s^{(1-n)} \qquad (17)$$

The value (1-n) varies as follows:

1-n = 0.71 - 0.77 for plate diffusers with full floor coverage
0.78 for 4 nozzle spargers with centerline header
0.45 for tube diffusers with one side header
0.8 - 1.0 for small orifice diffuser

King (1955) independently derived equations based upon his experimental observation, which showed the rate of oxygen absorption varied with (0.825 - 0.86) power of air fow rate, depending on liquid depth and geometry, using bench scale experimental facilities.

D-2. LIQUID DEPTH

The value of mass transfer coefficient is also affected by liquid depth, due to different surface renewal properties during bubble formation, rise, and burst. Generally, it decreases with increasing liquid depth above a submerged diffuser (Eckenfelder, 1959b; Eckenfelder and Barnhart, 1961; Khudenko, 1979)

Downing et al.(1962) presented their experimental results in terms of volume of oxygen absorbed in unit volume of water per unit dissolved oxygen (DO) deficit, as shown in Figure 3. Since the value of oxygen absorption per unit DO deficit was independent of the saturation concentration, this figure implied K_La values increased with liquid depth.

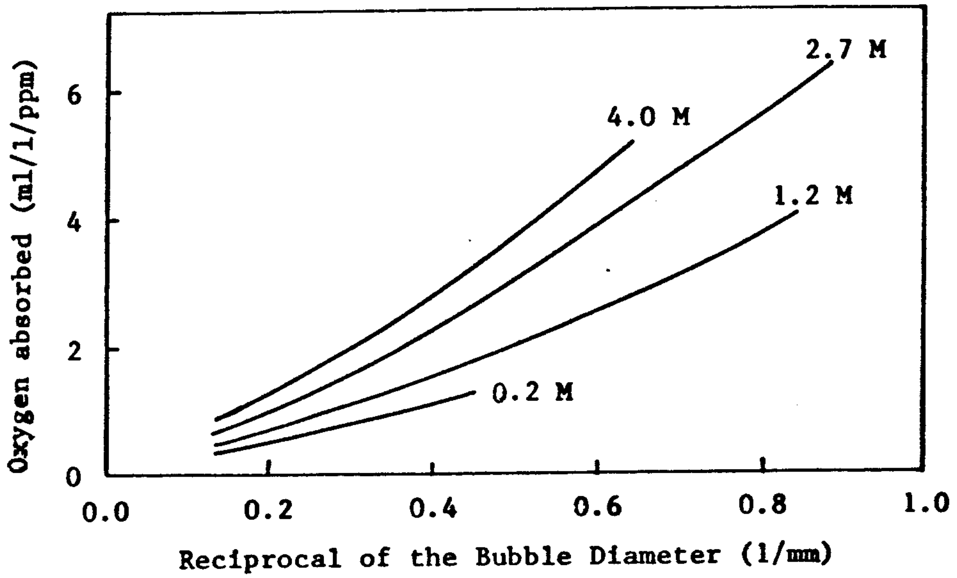


Figure 3. Oxygen Absorption vs. Reciprocal of Bubble Diameter

(After Downing et al., 1962)

D-3. HEADER AND DIFFUSER LOCATION

In the full size aeration tank, the aeration device lay-outs and the resulting flow patterns are important factors. For surface mechanical aeration devices, which are usually evenly distributed over the surface area, the distance between aerators and the presence and submergence of draft tubes can make large variations of overall mass transfer rate. A wide variety of references are available on this subject (Eckenfelder et al., 1962; Price et al., 1973), and are not dealt with here. For diffused aeration, the effects of diffuser lay-outs and flow patterns are presented in Table 1 after Von der Emde (1968) and Rooney and Huibregtse (1980).

Bewtra and Nicholas (1964) investigated the effect of tank widths and concluded that reducing tank size from 24' x 4' to 8' x 4' at 14.4' water depth changed mixing pattern from a spiral flow to "column aeration". This effect increased the oxygen transfer efficiency 40 % and 60 % for coarse and fine bubble diffuser, respectively. They stated that the reduced section had a "stilling effect", reducing vertical liquid velocity.

In case of a single or double header, diffuser placement in a narrow or wide band also has an effect. Bewtra and Nicholas (1964) observed higher oxygen transfer efficiency with a wide band than a narrow band on a centerline header, while Eckenfelder and Barnhart (1961) did not notice any significant difference between those two types mounted at the side of the tank. This result may be due to the fact that the aeration zone can be spread-out at the center with wide band, while it is pushed

A.

Mixing Pattern	Oxygen Transfer Rate (mg/l/hr)	% Improvement
Side axial	34	----
Center axial	43	26
20 ft cross	51	50
10 ft cross	59	74

B.

Aerator Location	Oxygen Transfer Efficiency and % Improvement					
	Saran Tube		Brandol Tubes		Plastic boxes	
One Side	13.4	----	16.1	----	13.0	----
Both Sides	14.3	6.7	15.6	-3.1	15.8	21.5
Full Floor	18.6	39.0	18.4	14.3	----	----

A. After Rooney and Huibregtse (1980)

(Coarse bubble diffuser)

B. After Von der Emde (1968)

Table 1. Effect of Diffuser Location and Flow Pattern

to the wall regardless the type of band, if the difusers are located on the side. Other researchers who worked on the types of diffusers, and their placement patterns, reported the similar results (Leary et al., 1969; Aberley et al., 1974)

The last but the most important variable affecting the mass transfer rate for gas molecules is water quality, which includes the presence and concentration of surface active agents, such as detergents, dissolved solids and perhaps suspended solids. This subject is discussed in the next section.

E. EFFECT OF WATER QUALITY ON OXYGEN TRANSFER

The performance of aeration devices is usually analyzed under standard conditions and the results are applied to field conditions through three parameters; alpha, beta, and theta (Stenstrom and Gilbert, 1981). Standard conditions for aeration systems are defined as:

Temperature = 20°C
Pressure = 1 atmospheric pressure (14.7 psia)
Test liquid = tap water
Dissolved oxygen concentration = 0 mg/l

The oxygen transfer rates under the field conditions can be predicted by the following relation;

$$\mathrm{OTR} = \alpha \cdot \frac{\beta \cdot C^*_{\infty_{TW}} - C_L}{C^*_{\infty_{20}}} \cdot \theta^{(T-20)} \cdot \mathrm{SOTR} \qquad (18)$$

where OTR = oxygen transfer rate at field conditions

SOTR = oxygen transfer rate at standard conditions

$$= K_L a_{20_{TW}} \cdot C^*_{20_{TW}} \cdot V \qquad (19)$$

α = alpha factor

$$= K_L a_{WW} \;/\; K_L a_{TW} \qquad (20)$$

β = beta factor

$$= C^*_{\infty_{WW}} \;/\; C^*_{\infty_{TW}} \qquad (21)$$

θ = theta factor

$$K_La_T = K_La_{20} \cdot \theta^{(T-20)} \tag{22}$$

T = temperature (^oC)

C_L = dissolved oxygen concentration in the field

There have been some questions about the validity for tap water as a reference liquid, and Baillod and Brown (1983) summarized the problems associated with the quality of tap water, and implied that several constraints on tap water would be needed changing, such as sulfate or total dissolved solid concentration. If a reference water quality were established other than "tap water", such as "distilled water", it would be uneconomical and impractical in case of full scale tests, such as 0.45 million gallon tests by Conway and Kumke (1966), 75,000 gallon tests by Yunt et al. (1982), and 1.0 million gallon tests by Yunt (1983).

The various parameters will be discussed in the following sections.

E-1. ALPHA FACTOR

Alpha factor has been defined as the ratio of the volumetric mass transfer coefficient in the wastewater to that in the tap water at equivalent conditions of temperature and oxygen partial pressure as follows:

$$\alpha = \frac{K_La_{WW}}{K_La_{TW}} \tag{20}$$

The alpha factor varies with many process conditions (Stenstrom and Gilbert, 1981, Hwang and Stenstrom, 1979) including method of aeration, intensity of mixing or turbulence, water quality, solids concentration, shape and scale of experimental facilities, and other unknowns.

The value of alpha factor shows greater variation with the diffused aeration than any other method, and the most severe effect is usually observed with the fine bubble diffuser. Kessener and Ribbius (1935) provided one of the earliest reference on the alpha values. Using their concentration versus time data, mass transfer coefficients were calculated (Stenstrom and Gilbert, 1981) to give 0.2 of alpha value for sterilized sewage using diffused aeration.

Oxygen mass transfer coefficients greatly depend on water quality and the air-water interfacial conditions. The presence of surfactants in water has two distinct effects on air-water interface:

(1). Small amounts of surface active agents reduce the surface tension dramatically, which results in a decrease in diameters of air bubbles or water droplets, and thereby increase the ratio of surface area to air or water volume.

(2). Accumulation of surfactant molecules at the air-water interface forming a "blanket" which reduces internal circulation of bubbles.

In the mechanical surface aeration method, at low power input level, the alpha value is much less than unity (Downing and Truesdale, 1955; Gameson et al., 1956; Downing et al., 1957; Stenstrom and Hwang, 1979), but with increasing power input, the alpha value approaches unity and sometimes exceeds unity at very high power input (Baars, 1955;

Eckenfelder and Ford, 1968; Stenstrom and Hwang, 1979). This change is due to the translation of predominant surfactant effect from the second-type to the first-type with increasing power input.

In the diffused aeration method, the second effect is predominant. The adsorbed molecules make the bubble more rigid and spherical (Baars, 1955), and lower the renewal rate of new air-water interface. As a result, alpha value under the presence of SAA is almost always less than unity in diffused aeration system. As discussed in the previous section, the effect of a cap is most significant when the bubble motion is either spiral or zig-zag, which occurs in the fine bubble aeration systems. Hence the alpha value is much more reduced in the fine bubble system than in the coarse bubble system.

Various researchers have reported a wide range of experimentally determined values of alpha factor. Holroyd and Parker (1952) reported alpha values of 0.63, 0.54, and 0.41 in the presence of 50 mg/l of anionics, Teepol, Lissapol, and Turkey Red Oil (British surfactants), respectively, in a bench scale diffused aeration, and 0.83, 0.93 and 0.85 with a 30 cm diameter disc surface aerator.

Baars (1955) reported that 10 mg/l of alkyl aryl sulphonate, alkyl sulfate and Lissapol reduced oxygenation capacity to 56.6%, 53.3% and 42.6% of that in tap water, respectively, in bench scale experiments using dome diffuser. Hwang and Stenstrom (1979) showed that alpha varied with 1.85 power of (σ_{WW} / σ_{TW}) in the range of 0.75 and 1.0 of (σ_{WW} / σ_{TW}) value using a laboratory scale diffused aeration system. Carver reported that 50 mg/l of synthetic detergent reduced oxygen transfer rate by 60 % for rising air bubbles (Carver, 1956) and 50 % for falling water droplets (Carver, 1969).

Alpha factor is also affected by solids concentration. Baker et al. (1975) reported that alpha factor decreased from 0.9 to 0.4 when total solids concentration of poultry waste was increased from 1 to 5.5% in a oxidation ditch using cage rotors. Downing (1960) found that only a slight increase in K_La as suspended solids concentration increased in the range of 2000 to 7000 mg/l, while Lister and Boon (1973) reported no variation of K_La in the same range. Poon and Campbell (1967) reported that sodium chloride of up to 4000 mg/l of concentration had almost no effect on alpha value, but soluble starch and glucose reduced alpha to 0.7 at the same concentrations, while peptone and nutrient broth gave approximately 0.5 of alpha value at 1000 mg/l using a fritted glass diffuser in a 5.5 in x 7 ft water column.

Holroyd and Parker (1952) reported essentially no effect on K_La values with the addition of bentonite, a highly colloidal clay, using bench scale diffused aeration devices. Casey and Karmo (1974) obtained similar results with coarse and fine dried peat granules and PVC granules in concentrations up to 10 g/l, from the laboratory scale experiments of surface aeration. However, they also reported that alpha factor varied from 0.84 to 1.2 with the concentration of activated sludge over the range of 0 to 12.5 g/l. Jackson and Hoech (1977) reported that an alpha value of 0.69 for water containing 600 ppm groundwood pulp which increased to 0.72 after filtering with a Buchner funnel and Whatman No. 41 filter paper.

Water Depth (m)	Symbol
3.7	○
6.1	△
8.1	□

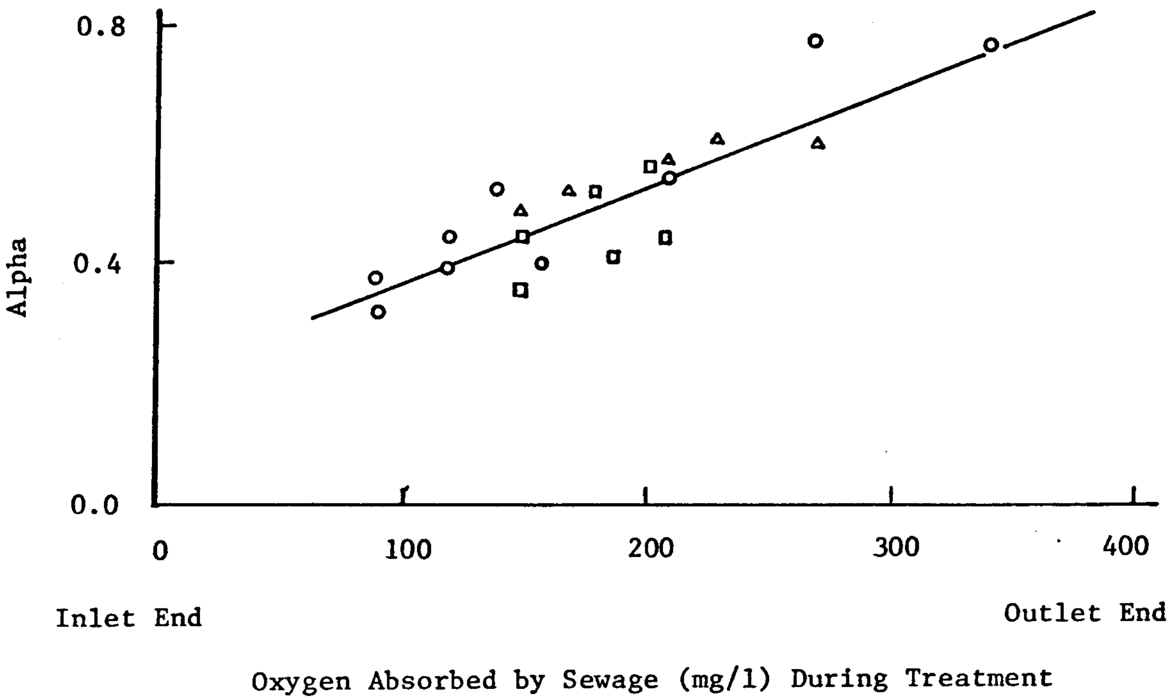


Figure 4. Alpha vs. Oxygen Consumed (mg/l)

(After Wheatland and Boon, 1979)

From these reports, it is clear that alpha factor depends primarily on dissolved solids rather than total or suspended solids concentration. This also implies that alpha factor will depend upon degree of treatment, since treatment can remove much of the organic dissolved solids. This implication has been verified by Downing et al. (1962) and Wheatland and Boon (1979), who reported the linear dependence of alpha value on the amount of oxygen consumed by the microorganisms in a "plug-flow" aeration basin, as shown in Figure 4.

Similar results were reported by Redmon and Boyle (1981) with alpha increasing from 0.35 to 0.6 in tests using off-gas method in the full scale fine bubble diffused aeration facility in Whittier Narrows Plant of L.A. County. In batch studies conducted by Doyle (1981), alpha factor increased with time as shown in Figure 5.

Alpha also varies with the scale of experimental facility which primarily depends on water height above diffuser and mixing intensity.

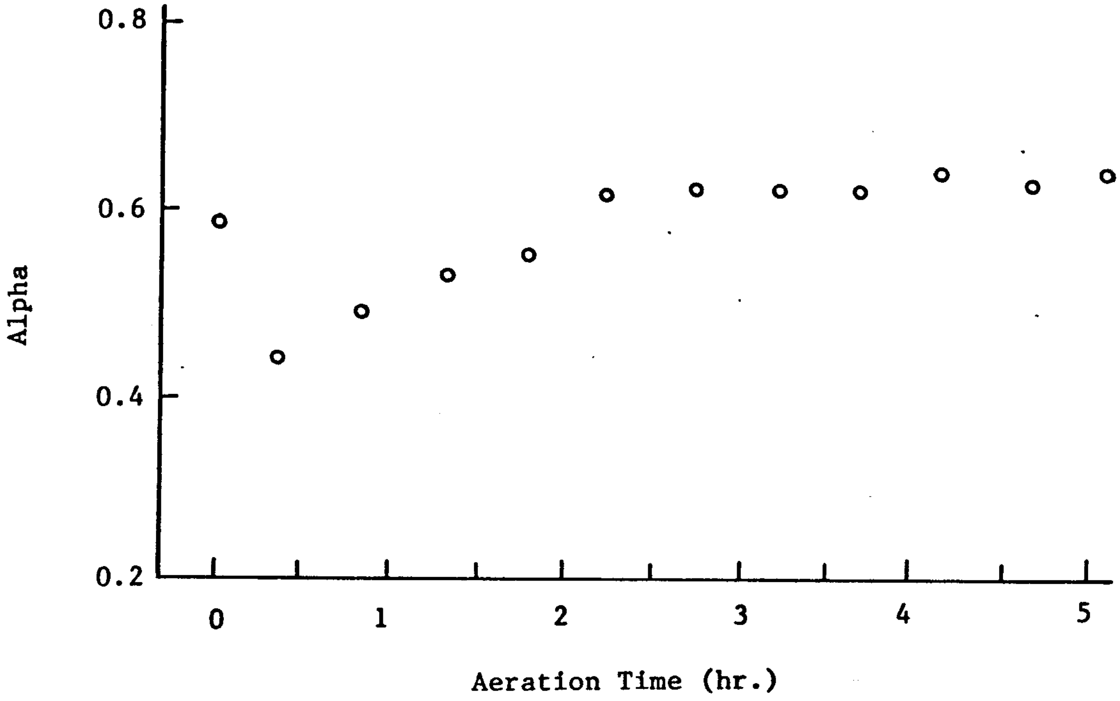


Figure 5. Alpha vs. Aeration Time

(After Doyle, 1981)

E-2. BETA AND THETA FACTORS

Oxygen saturation concentration decreases with increasing ionic strenth, and correlations of saturation value with total dissolved solids or chloride concentration have been made (APHA, 1980).

The beta factor has been defined as the ratio of the saturation dissolved oxygen concentration in wastewater to that in tap water at equivalent conditions of temperature and oxygen partial pressure, as follows:

$$\beta = \frac{C^*_{\infty_{WW}}}{C^*_{\infty_{TW}}} \tag{21}$$

Stenstrom and Gilbert (1981) discussed the technical problems measuring beta factors using either Winkler test or DO probe and recommended an alternative method proposed by Bass and Shell (1979) which uses an analytical correction factor based upon total solids concentration. The beta factor, as defined above, does not account for the effect of temperature, oxygen partial pressure, and relative humidity. when the saturation concentration at field condition is estimated from oxygen solubility data in water, the value must be corrected for those parameters mentioned above as well as total dissolved solids concentration. The detail can be found in ASCE Report (Stenstrom and Gilbert, 1981).

Beta factors have been found to vary over a wide range, but the range is much less than that of the alpha factor. Tewari and Bewtra (1982) reported that beta balue varied from 0.96 to 0.98 in four different mixed-liquor samples, while alpha variation was from 0.67 to 0.925. Eckenfelder et al.(1956) reported beta value of 0.95 for domestic wastewater which is a generally accepted value, although this is less than what one would expect from the ionic strenth of wastewater.

Theta factor has been defined to compensate temperature effect on volumetric mass transfer coefficient for equivalent conditions of turbulence, oxygen partial pressure, and water quality, as follows:

$$K_La_T = K_La_{20}\cdot\theta^{(T-20)} \tag{22}$$

Theta factors have been reported to vary from 1.008 to as high as 1.047, and some researchers believe that theta factor is a function of turbulence (Stenstrom and Gilbert, 1981), but at present, there is not sufficient information to include such variables in a design standard. Theta value of 1.024 is generally accepted, in lieu of an experimentally defined value. No correction for temperature on saturation concentration value is required, since the effect of temperature is explicitly known.

F. NONDIMENSIONAL ANALYSIS OF SURFACE AERATION

The major difficulty in predicting performance of full scale equipment is "scale-up". Some authors doubt the validity of the scale-up procedure. According to Fisherstrom (1960), it is not useful to establish experimental purpose models corresponding to a size ratio of less than 1/2, and for economic evaluation, he recommends only full scale systems, after analyzing the performance of Inka type air injection system. However, the need for scaling results from testing at small scale, which is often the only economical alternative, and many researchers have reported reasonable success in the scale-up process, especially with mechanical surface aerators.

Successful scaling-up requires similarity criteria between model and prototype, such as geometric, dynamic and environmental similarity. Schmidtke (1982) further extended these requirements to microorganisms and the floc particles. If these criteria are satisfied, one should be able to express the mass transfer rate in explicit or implicit form with the nondimensional parameters.

Nondimensional Parameters

Schmidtke (1982) presented the following variables influencing fluid motion in an unbaffled, surface turbine agitated, square tank, as follows:

$$f(W, D, HI, WI, LI, B, S, \rho, \nu, g, n, P) = 0 \qquad (23)$$

where W = aeration tank width

D = liquid depth

HI = impeller blade height

WI = impeller blade width

LI = impeller blade length

B = number of impeller blades

S = impeller pitch

ρ = fluid density

ν = fluid kinematic viscosity

g = gravitational constant

n = impeller speed

P = power input

and for oxygen transfer, the corresponding variables are:

$$f(K_La, DI, HI, n, \nu, D_L, g, \rho, \sigma, P) \qquad (24)$$

where DI = impeller diameter

D_L = diffusivity coefficient for oxygen in water

σ = surface tension

From these sixteen variables, thirteen nondimensional parameters can be derived as:

$$Ff\left(\frac{HI}{DI}, \frac{W}{DI}, \frac{W}{D}, \frac{WI}{DI}, \frac{LI}{DI}, \frac{S}{DI}, B, Re, Fr, Ne, \frac{K_L a}{n}, Sc, We\right) = 0 \qquad (25)$$

where Re = Reynolds number

$$= \frac{n \cdot DI^2}{\nu} \qquad (26)$$

Fr = Froude number

$$= \frac{n^2 \cdot DI}{g} \qquad (27)$$

Ne = Power number

$$= \frac{\rho \cdot n^3 \cdot DI^5}{P} \qquad (28)$$

Sc = Schmidt number

$$= \frac{\nu}{D_L} \qquad (29)$$

We = Weber number

$$= \frac{\rho \cdot n^2 \cdot DI^3}{\sigma} \qquad (30)$$

Geometric similarity requires constant values of first through seventh variables, and using tap water as test liquid, Sc, We can be kept constant. In the domain of high turbulence, Re is independent of the oxygen transfer and by keeping Ne constant, $K_L a/n$ can be only function of Fr. Based upon these assumptions, Schmidtke proposed the following relation.

$$K_L a = a \cdot n^b \cdot DI^c \tag{31}$$

where a, b, and c are experimental constants

When volumetric mass transfer coefficients have same value in model and prorotype, the scale-up criteria can be:

$$n_m = n_p \cdot \left(\frac{DI_p}{DI_m} \right)^d \tag{32}$$

where subscripts m = model

p = prototype

d = experimental constant

From the data obtained with four different size experimental facilities varying from bench scale to laboratory scale, he determined the values as 0.65.

Zlokarnik (1979) simplified the geometric parameters to only the diameter of the different types of surface aerator, and four material parameters for liquid; ρ, ν, σ, and D_L and two process parameters, n and g. Then he derove five nondimensional groups, as follows:

$$Y = f(Re, Fr, Sc, We) \tag{33}$$

where Y = transfer number

$$= \frac{G}{\Delta C \cdot DI^3} \cdot \left(\frac{\nu}{g^2} \right)^{1/3} \quad (34)$$

$\Delta C = C_\infty^* - C_L$

G = mass transfer rate

$$= K_L a \times C_\infty^* \quad (35)$$

He related oxygen transfer and power numbers to Froude number to derive following relation after four types of rotors.

$$Y \cdot Ne^{-1} \cdot Fr^{-3/2} = \text{constant} \quad (36)$$

III. DIFFUSED AERATION COLUMN

A. EXPERIMENTAL METHODS

A-1. EXPERIMENTAL METHODS IN CLEAN WATER

The performance of an aerator can be evaluated by either steady-state or nonsteady-state method in terms of dissolved oxygen concentration. In clean water, the nonsteady-state method is applicable. Some researchers have applied a steady-state method in clean water by oxidizing excessive concentration of sulfite ion in solution (Dreier, 1955; Urza and Jackson, 1975), but the mass transfer coefficient obtained from this procedure is enhanced by chemosorption (Linek and Benes, 1978). Conway and Kumke (1966) reported that sulfite ion depletion method gave 44 % higher result than nonsteady state reaeration method.

For the nonsteady-state test, the dissolved oxygen level is lowered to zero before the test by oxidizing added sodium sulfite ions with cobalt ions as a catalyst, or by stripping oxygen molecules out with nitrogen gas. The water is then reaerated to the saturation concentration.

A-2. EXPERIMENTAL METHODS IN RESPIRING SYSTEMS

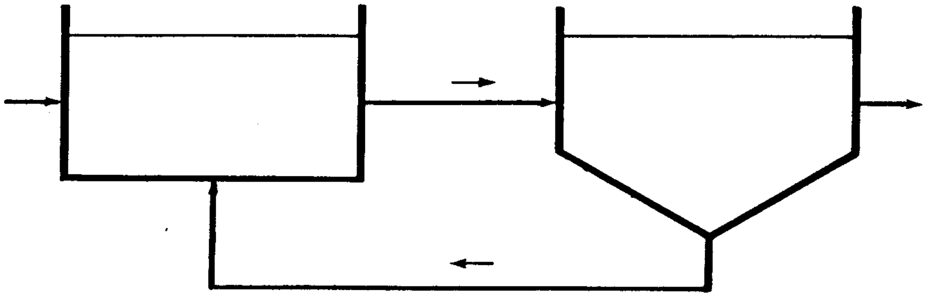
Due to difficulty of modeling the variability of biological oxygen consumption, it is a common practice to determine alpha factor in wastewater from which the most of the biological solids are removed, e.g., clarified activated sludge effluent (Stukenberg et al., 1977), or in which the bioactivity is terminated by adding 1 g/l of copper sulfate (Bass and Shell, 1977), or in which the bioactivity is in endogenous phase (McKinney, 1981). As discussed in section II-E-1, the alpha value determined under those conditions does not represent the value under process conditions.

Two basic procedures for testing respiring systems have been developed:

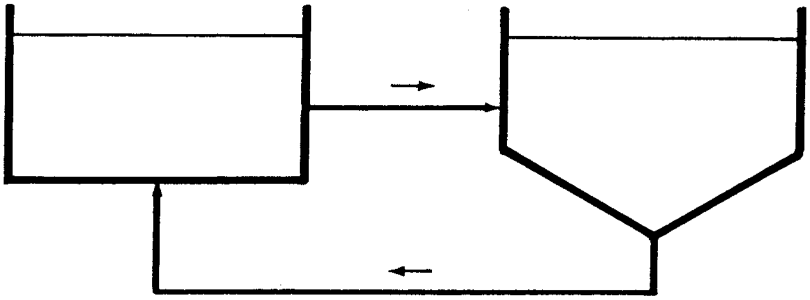
- Continuous Flow System
- Batch System

Both systems have steady state and nonsteady state procedures, in terms of dissolved oxygen concentration. All methods have well defined procedures to obtain values of K_La and C^*_∞ , but unfortunately these procedures require large experimental facilities and many variables must be controlled and measured.

Another recently described method is the so-called "off-gas method", which is based on an oxygen mass balance in entering and exiting gas, rather than that in liquid phase. This test method is suitable under normal operating conditions, however, it does not directly measure the oxygen saturation concentration. Therefore, it is necessary to estimate



(a) Continuous Flow System



(b) Batch System

Figure 6. Schematic Diagram of Mixed Liquor Test

the saturation concentration through Henry's constant and beta factor. The off-gas procedure requires a special hood to collect the off-gas from the reactor and extremely accurate oxygen partial pressure sensing devices. Furthermore, this method is only applicable to bubble aeration systems.

A promising method appears to be a simple batch column with suitable diameter to minimize wall effects and a height similar to that of full scale aeration tank under operation. Using this technique, one investigator showed that the alpha value is in the range of 0.4 and 0.9 for domestic wastewater in 12-inch diameter aeration column (Doyle, 1981).

All procedures discussed above fail to account for the rapid change of water quality and resulting variation of volumetric mass transfer coefficient and oxygen saturation concentration.

B. MATHEMATICAL MODELS

B-1. CLEAN WATER NONSTEADY STATE BATCH TEST

Equation 3 for the gas transfer through the gas-liquid interface is usually modified for the analysis of both surface and diffused clean water, nonsteady state oxygen transfer data, as follows:

$$\frac{dC}{dt} = K_L a \cdot (C_\infty^* - C) \tag{37}$$

where $K_L a$ = volumetric mass transfer coefficient

C_∞^* = average dissolved oxygen concentration attained at infinite time

C = effective average DO concentration in the liquid phase.

which can be integrated to become:

$$C = C_\infty^* - (C_\infty^* - C_0) \cdot \exp(-K_L a \cdot t) \tag{38}$$

where C_0 = initial dissolved oxygen concentration

B-2. CONSTANT OUR NONSTEADY STATE BATCH TEST

In a simple batch reactor, the oxygen mass balance equation in the liquid phase can be written as follows:

$$\frac{dC}{dt} = K_La\ (C_\infty^* - C) - R(t) \tag{39}$$

When the microbial oxygen uptake rate can be assumed constant with time, as in the endogenous phase, equation 39 becomes:

$$\frac{dC}{dt} = K_La\ (C_\infty^* - C) - Rc \tag{40}$$

where R_c = time-invariant microbial oxygen uptake rate

This ordinary differential equation can be solved as:

$$C = C_R - (C_R - C_0)\ \exp(-K_La\ t) \tag{41}$$

where

$$C_R = C_\infty^* - \frac{Rc}{K_La} \tag{42}$$

B-3. EXPONENTIALLY DECAYING OUR NONSTEADY STATE BATCH TEST

The microbial oxygen uptake rate in the respiring system is dependent on the substrate concentration, microorganism population and activity in the system, temperature, pressure, turbulence, and the other factors. For a short period of time, it can be expressed as follows, if other process conditions are held constant:

$$R(t) = a \times L(t) + b \times S(t) \tag{43}$$

where $R(t)$ = microbial oxygen uptake rate (the mass of oxygen utilized by microorganisms per unit time and unit volume)

$L(t)$ = the mass concentration of soluble substrate at time t

$S(t)$ = mixed-liquor suspended solid concentration

a, b = proportionality constants

The first term represents oxygen consumption due to substrate metabolism, while the second term represents the oxygen consumption due to endogenous respiration. Under process conditions the rate of biological oxidation of substrate is usually expressed by a first-order reaction as follows:

$$\frac{dL(t)}{dt} = - Ku \cdot L(t) \tag{44}$$

Then substrate concentration is expressed after integration, as:

$$L(t) = L_0 \cdot \exp(-Ku \cdot t) \qquad (45)$$

where L_0 = initial substrate concentration

If the variation of sludge solids concentration with time can be neglected, the second term in the right-hand side of equation 43 can be considered to be constant and the total oxygen uptake rate can be rewritten as:

$$R(t) = a \cdot L_0 \cdot \exp(-Ku \cdot t) + b \cdot S(t) \qquad (46)$$

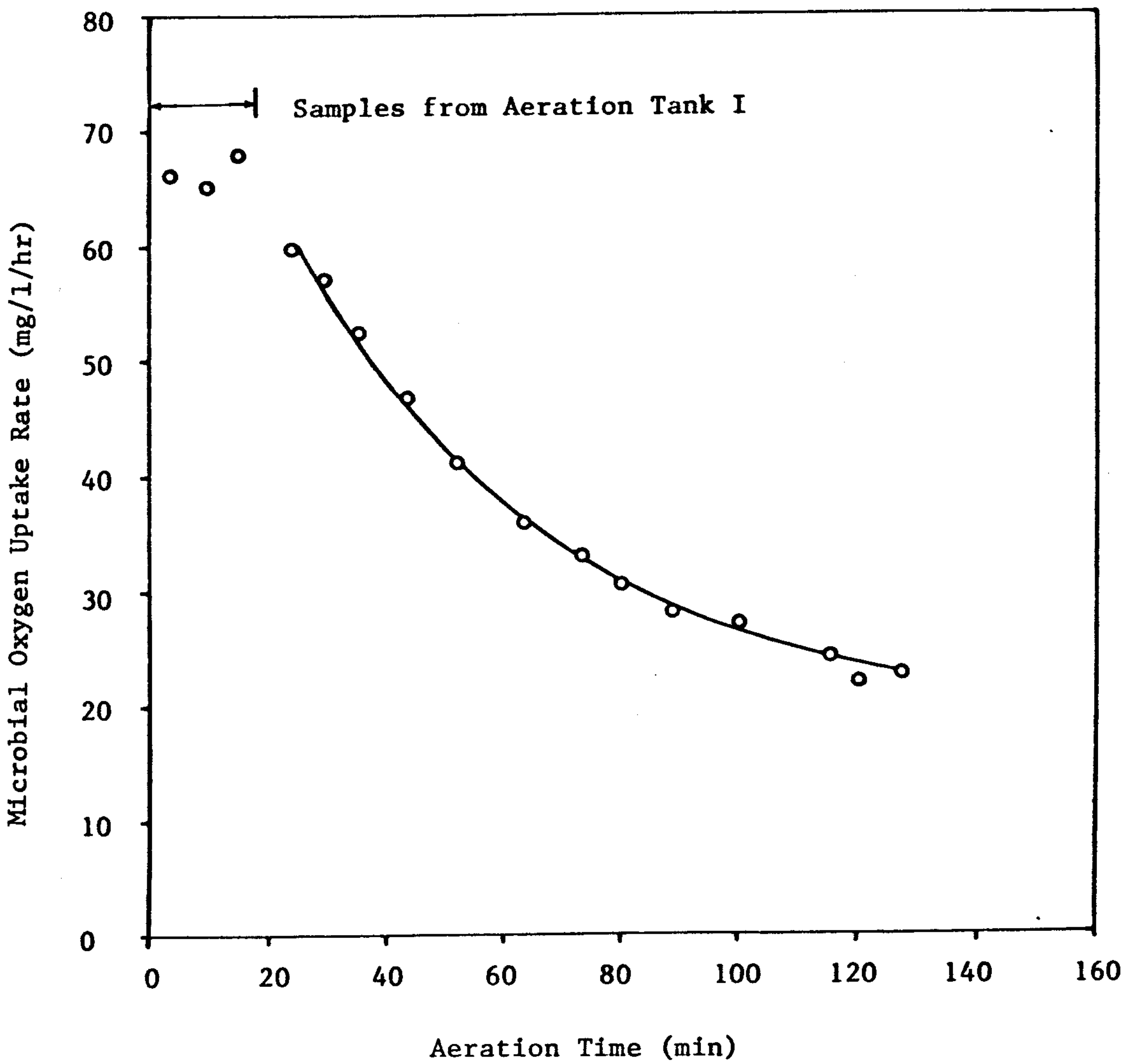
$$= R_0 \cdot \exp(-Ku \cdot t) + R_c \qquad (47)$$

where R_0 and R_c are constants

Figure 7 shows the experimental data fitted to equation 47.

An assumption underlying this derivation is that the microbial oxygen uptake rate is independent of dissolved oxygen concentration over the range encountered in the experiment. Gaden (1955) showed that oxygen uptake rate was independent of oxygen concentration above a critical concentration. Below the critical concentration, the reaction is approximately first order. The reported critical values in mixed-liquor are summarized in Table 2.

The dissolved oxygen concentration can be expressed by combining equations 39 and 47, as follows:



(4/7/82, 2nd fill of aeration column,

mixed-liquor from 15 ft location of tank I)

Figure 7. Oxygen Uptake Rate (mg/1/hr) vs. Time (min)

Reference	Critical Concentration
Eckenfelder and O'Connor, 1955	0.2 - 0.5 mg/l
Gaden, 1955	0.6
Busch, 1970	0.5
Bailey and Ollis, 1977	0.1 - 0.7 *
Stukenberg et al, 1977	0.6

* range for different microorganisms except for Azotobacter which require 0.6 - 1.6 mg/l

Nitrifing systems may require higher DO concentrations

Table 2. Critical Dissolved Oxygen Concentration for Bioactivity

$$\frac{dC}{dt} = K_La \cdot (C^*_\infty - C) - (R_0 \cdot \exp(-Ku \cdot t) + R_c) \tag{48}$$

If K_La can be assumed constant, this odinary differential equation can be solved to give following expression:

$$C = C_R - (C_R - C_0 - \frac{R_0}{K_La - Ku}) \cdot \exp(-K_La \cdot t) - \frac{R_0}{K_La - Ku} \cdot \exp(-Ku \cdot t) \tag{49}$$

As the wastewater is treated, the mass transfer coefficient increases (Downing et al., 1962; Wheatland and Boon, 1979). At the entrance to the aeration tank, which compares to the period soon after a batch aeration column is filled with fresh mixed-liquor, the substrate concentration and oxygen uptake rates are high, while the mass transfer coefficient is very low, which results in a longer period of aeration time to reach a relatively high dissolved oxygen concentration. Actually in the initial phases of an experiment, the steady state condition often cannot be achieved due to decreasing oxygen uptake rate by microorganisms and increasing oxygen transfer rate from air to water. Under these conditions, the assumption of a constant oxygen transfer rate in equation 48 would result in a significant error, and is clearly shown in the data of Doyle (1981). The estimation error of K_La value would also result in an error in the estimation of saturated value as indicated by the data.

In the batch test of mixed-liquor, the properties changing with time are suspended and dissolved solid concentrations, surface tension, viscosity, substrate and biomass concentrations, microbial oxygen uptake

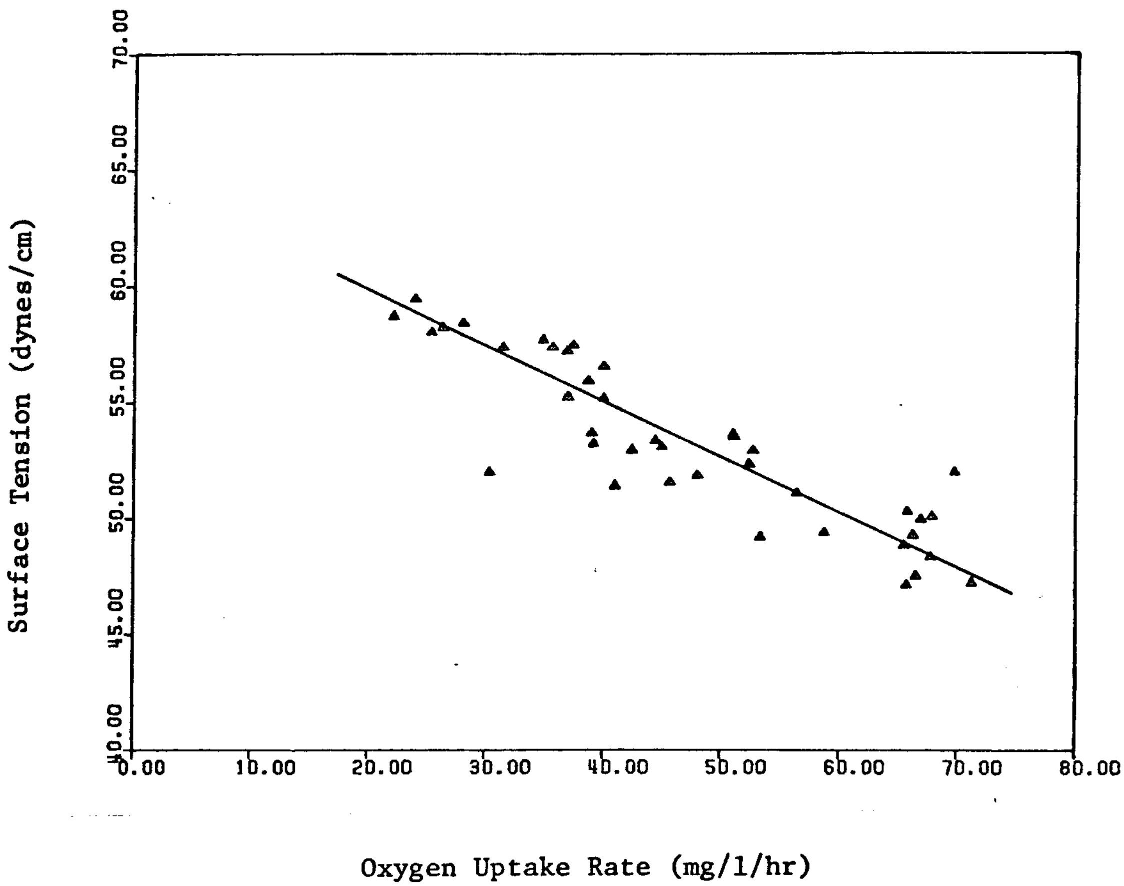


Figure 8. Surface Tension (dynes/cm) vs. Oxygen Uptake Rate (mg/l/hr)

rate, and temperature. Among these variables, variations of suspended solids, biomass concentration and viscosity are not significant. Futhermore, temperature does not change more than $1^{\circ}C$ per hour under usual experimental conditions. Surface tension will increase along with a reduction of surface active agents and dissolved organics concentration. A decrease in oxygen uptake rate will result from the decrease of substrate or dissolved organics concentration, as the system goes toward the endogenous phase. As expected, experimental data shows that surface tension and uptake rate data have a high correlation (Figure 8). From the discussion above, the parameters involved in oxygen transfer can be correlated to one variable, oxygen uptake rate. This variable can be measured quickly, accurately and on site.

The mass transfer coefficient in the batch of mixed liquor does not increase linearly with time, but its increase is rapid at the initial stage and decreases with time as the mixed-liquor is treated. After a sufficient time of aeration, the mass transfer coefficient reaches a steady state value of endogenous phase. Therefore, the change of K_La can be expressed with a mathematical equation, such as power function, exponential function, or Monod type function. The exponential function has an advantage over other forms, since the oxygen uptake rate has the same expression:

$$K_La = K_La_0 \cdot \exp(-Kk \cdot t) + K_La_c \qquad (50)$$

Rearranging equations 47 and 50 give the following expression:

$$K_La \;=\; K_La_0 \cdot R_0^{-Kk/Ku} \cdot (R(t) - R_c)^{Kk/Ku} + K_La_c \qquad (51)$$

If Kk/Ku close to unity, K_La can be expressed as:

$$K_La \;=\; K_La_0 \cdot R_0 \cdot (R(t) - R_c) + K_La_c \qquad (52)$$

$$= \; a_c \cdot R(t) + b_c \qquad (53)$$

$$\text{where} \quad a_c = K_La_0 \cdot R_0 \qquad (54)$$

$$b_c = - K_La_0 \cdot R_0 \cdot R_c + K_La_c \qquad (55)$$

Even when the value of Kk/Ku deviates from the unity, for the short period of time, less than an hour during nonsteady state test, linearity assumption does not give significant error. This linearity assumption can be checked by a series of experiment over relatively broad range of oxygen uptake rates for the given air flow rate or mixing intensity.

Then the mass balance equation becomes:

$$\frac{dC}{dt} \;=\; (a_c \cdot R(t) + b_c) \cdot (C_\infty^* - C) - R(t) \qquad (56)$$

and

$$R(t) \;=\; R_0 \cdot \exp(-Ku \cdot t) + R_c \qquad (47)$$

Since the oxygen uptake rate is independent of dissolved oxygen concentration when dissolved oxygen concentration is above critical concentration. Ku, Ro and Rc can be estimated from R(t) versus time data using equation 47, and a_c, b_c and C_∞^* values can be estimated using equation 56.

Equations 56 and 47 are valid only when the process operation is above the critical dissolved oxygen concentration. Otherwise, the biological activity is not independent of dissolved oxygen concentration. If testings were performed when the process was operated below critical concentration, the value of K_La obtained would be related to dissolved oxygen concentration, process loading rate, and other factors.

B-4. CONSTANT OUR NONSTEADY STATE CONTINUOUS FLOW TEST

The development of mathematical models so far is confined to the batch reactor. When the resipiring system is operated in continuous flow mode, the oxygen mass balance equation is

$$\frac{dC}{dt} = K_La \cdot (C_\infty^* - C) - R(t) - \frac{Q}{V} \cdot (C - C_i) \qquad (57)$$

where C_i = DO in incoming mixed-liquor

Q = Mixed-liquor flow rate

When microbial oxygen uptake rate is constant, equation 57 becomes

$$\frac{dC}{dt} = K_La \cdot (C_\infty^* - C) - R_c - \frac{Q}{V} \cdot (C - C_i) \qquad (58)$$

and the odinary differential equation can be solved to give

$$C = C_R' - (C_R' - C_0)\cdot\exp(-K_La'\cdot t) \tag{59}$$

where

$$K_La' = K_La + \frac{Q}{V} \tag{60}$$

$$C_R' = \frac{K_La}{K_La'}\cdot C_\infty^* - \frac{1}{K_La'}\cdot(R_0 - \frac{Q}{V}\cdot C_i) \tag{61}$$

When R (t) is not constant, the differential equation can not be solved analytically.

C. EXPERIMENTAL SET-UP

The experimental facility for diffused aeration column tests was assembled in the Whittier Narrows Wastewater Reclamation Plant (WNWRP) of Los Angeles County Sanitation Districts (LACSD), which was located 30 miles east of UCLA campus.

The plant consists of three primary sedimentation tanks, three long and narrow aeration tanks (30 ft W x 300 ft L x 16 ft D), six secondary sedimentation tanks, coagulation and filtration facilities, and chlorination tanks. This plant receives relatively constant flow of domestic wastewater from the main trunk sewers which flows to the Joint Water Pollution Control Plant (JWPCP) of LACSD in Carson, California. The chemical oxygen demand (COD) of primary effluent exhibits a marked diurnal variation. The minimum concentration occurs at around 7 a.m. and the maximum concentration remains relatively constant in the whole afternoon. This trend does not show significant variation from day to day.

This Whittier Narrows plant was selected for evaluation of aeration systems by LACSD and the U.S. Environmental Protection Agency (EPA), and each aeration tank was equipped with a different type of diffuser. The Figure 11 shows the aeration tank lay-out at the plant. At the time of this experiment, the first two tanks were equipped with fine bubble diffusers and the third with jet aerators. The disc diffusers in tank 1 were installed in a full floor configuration and the tube diffusers in tank 2 were installed near both side walls to create a cross roll. Both

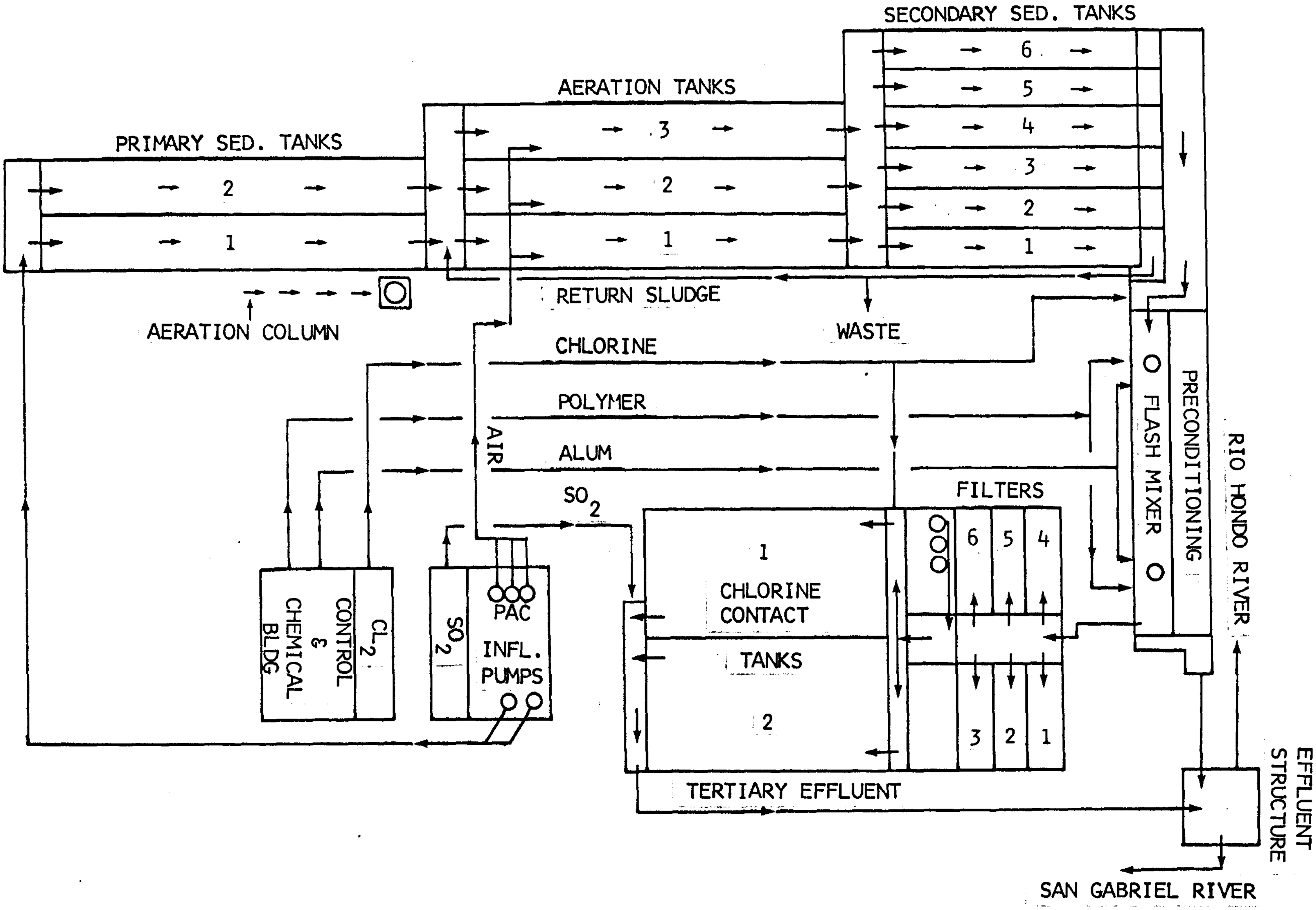


Figure 9. Whittier Narrows Wastewater Reclamation Plant

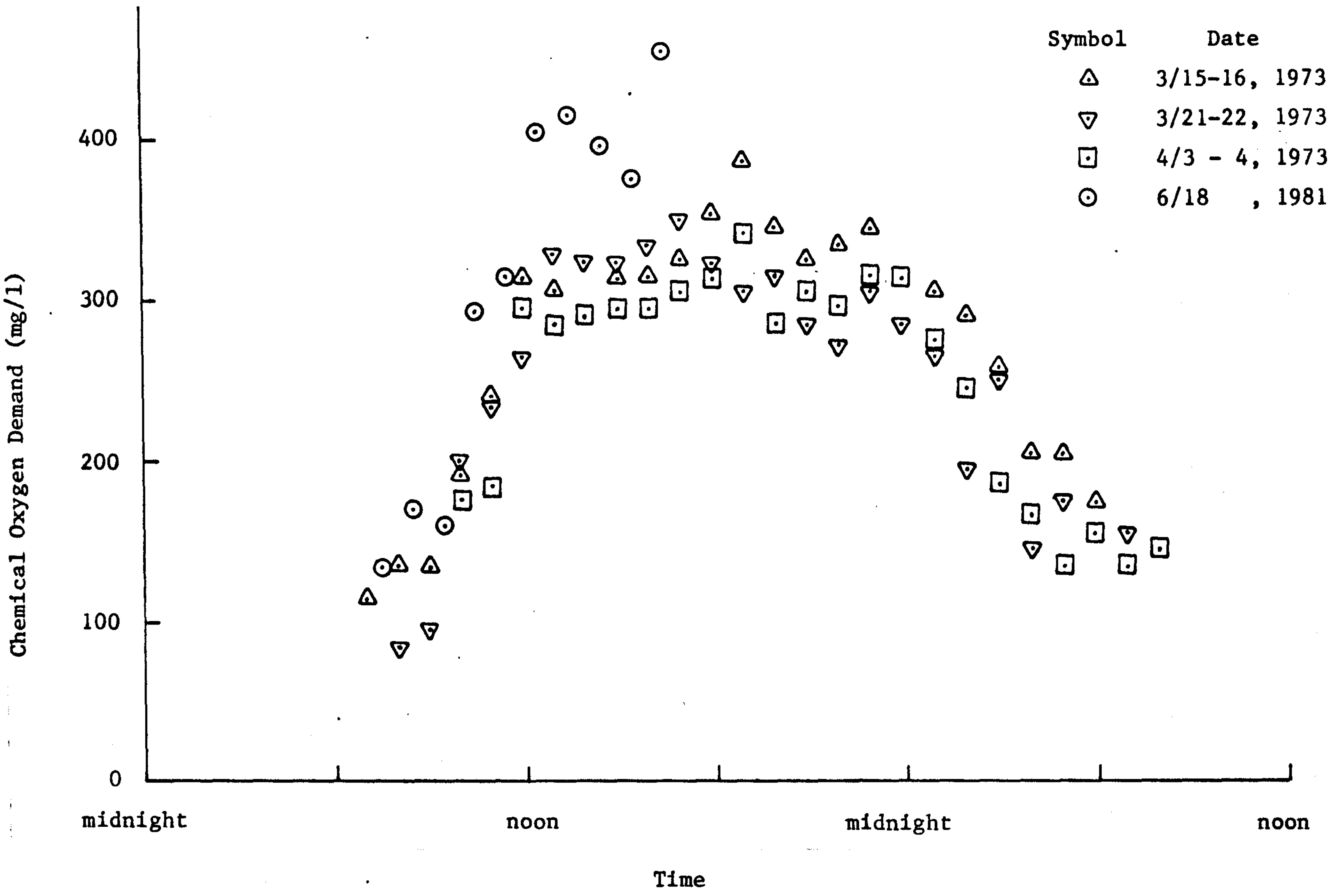


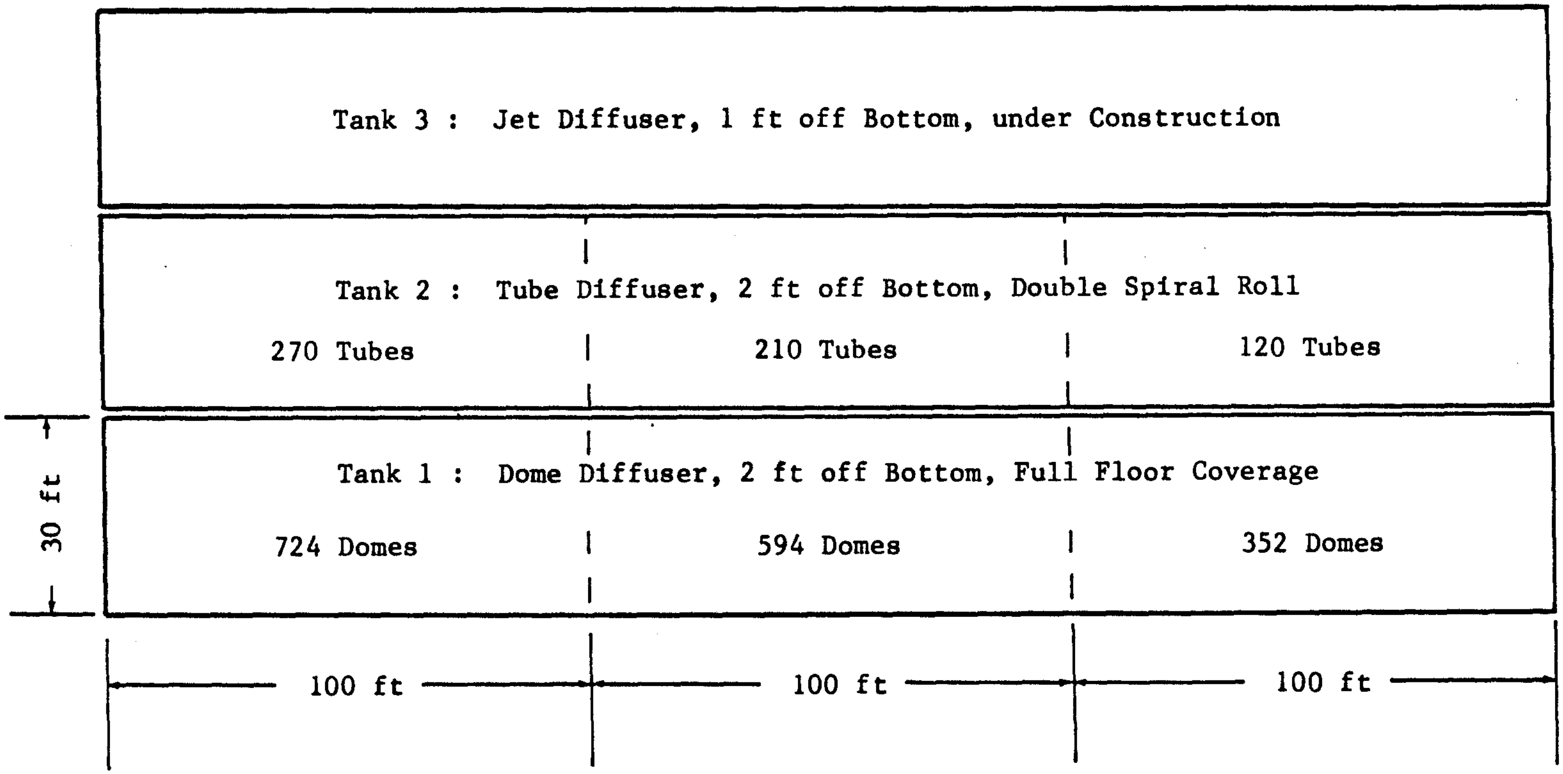
Figure 10. Variation of Chemical Oxygen Demand in WNWRP

diffusers were installed in a tapered aeration mode with approximately 45, 35 and 20 % of the total number of diffusers in each one-third of the tank from the inlet.

The preliminary study in the aeration tank 1 shows microbial oxygen uptake rate in the first grid changes markedly with influent COD concentration, but the oxygen uptake rate in the second grid is much less affected by influent COD variation, and the effect is less pronounced in the third grid. As shown in Figure 12 uptake rate in the first grid reduces from the value of grid 1 close to constant value and it is about one and one-half hours from the first grid. Those findings are the same trend with the report by McKinney (1981) and others.

The aeration column, a 16-foot long PVC pipe of 20-inch diameter, was set up on a unistrut base adjacent to the aeration tank 1. Taps were drilled at 2-foot intervals along the column for water depth and temperature measurements, water sampling for dissolved oxygen determination by Winkler tests, and microbial oxygen uptake rate measurements. A seven-inch diameter fine bubble diffuser plate (Sanitaire), was mounted on 1/2-inch PVC pipe and supported by two horizontal bars, and placed 2 feet above the column bottom, which was the same height as in the full scale aeration tank. Figure 13 shows the experimental set-up.

Dissolved oxygen concentration measurements were accomplished by submerging two DO probes (Yellow Springs Instrument, 5739) through the top of the column. Two DO meters (YSI 51B) and a continuous strip chart recorder (Houston Instrument B5217.5) were used to record dissolved



* Tank 1: Mixed Liquor Depth ; 14.3 ft, Flow Rate ; 6 MGD.

Figure 11. Aeration Tanks in WNWRP

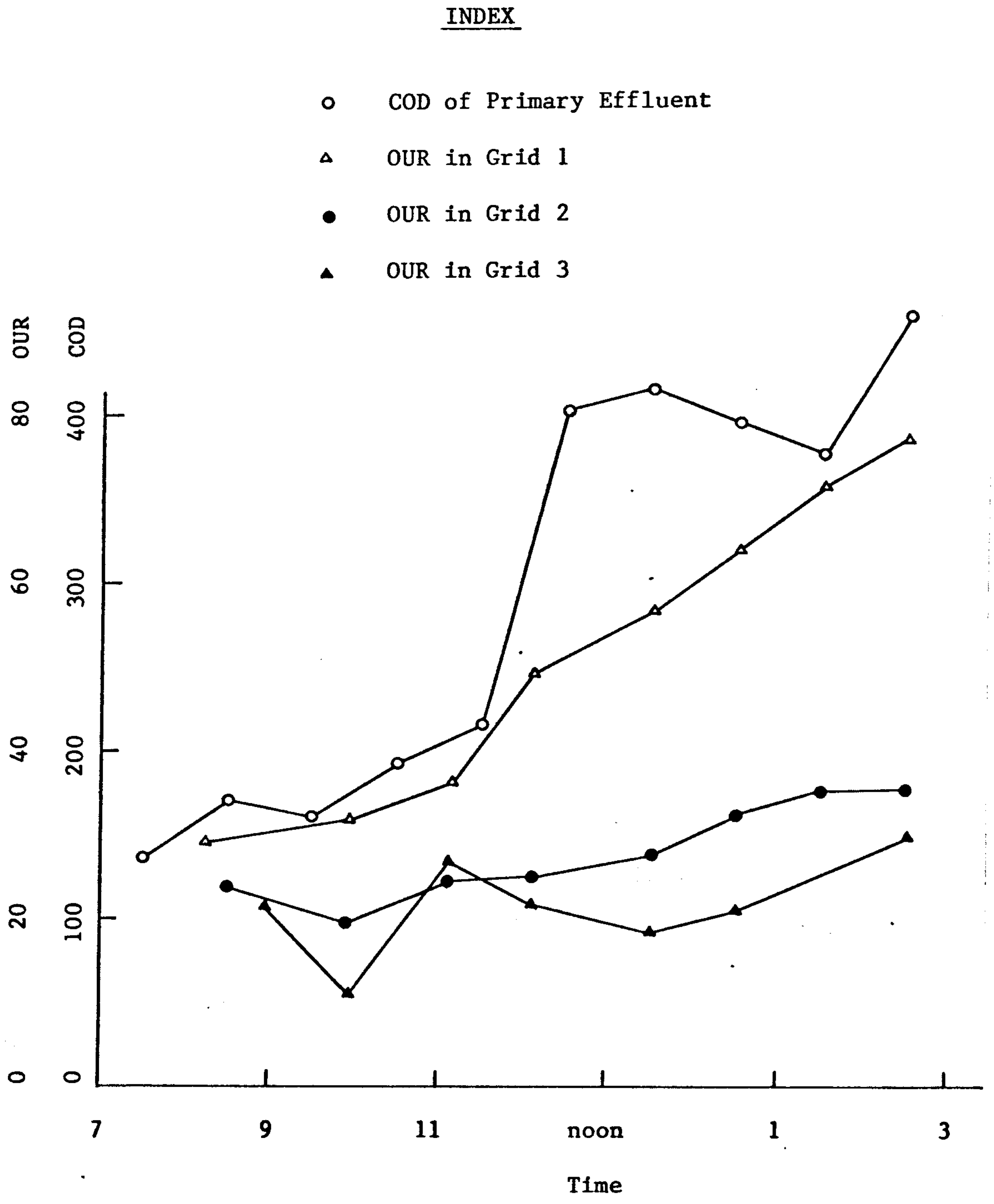


Figure 12. Microbial Oxygen Uptake Rate in WNWRP

oxygen concentration versus time data. To measure the microbial oxygen uptake rate for the mixed-liquor tests, an additional DO probe (YSI 5720A) and meter (YSI 57) were connected to Hewlett Packard Recording Digital Voltmeter (HP 3467a) which printed DO meter output, in millivolts, versus time data in 30 second intervals. The millivolt reading was converted to DO concentration using conversion factor established for each meter by Winkler method.

Two rotameters, three U-tube manometers and valves were mounted on a vertical panel and set beside the column to measure gas flowrate and various pressures. The rotameters used were Gilmont Flowmeter, Size No.5 which has maximum range of 3.0 scfm and Fischer-Porter of 1.1 scfm.

Atmospheric pressure was measured with the Prince Nova Mercury Barometer. Three pressure drops were measured:

p1 = pressure at compressed air after rotameter
(mercury column)

p2 = pressure drop across orifice of diffuser unit
(water column)

p3 = pressure drop across orifice and diffuser plate
(water column)

Various temperatures were measured with a mercury thermometer in the early series of experiments and later measured with thermocouples and D.C. voltmeters. The temperature measurements were as follows:

T1 = water temperature measured at approximately
10 minute intervals

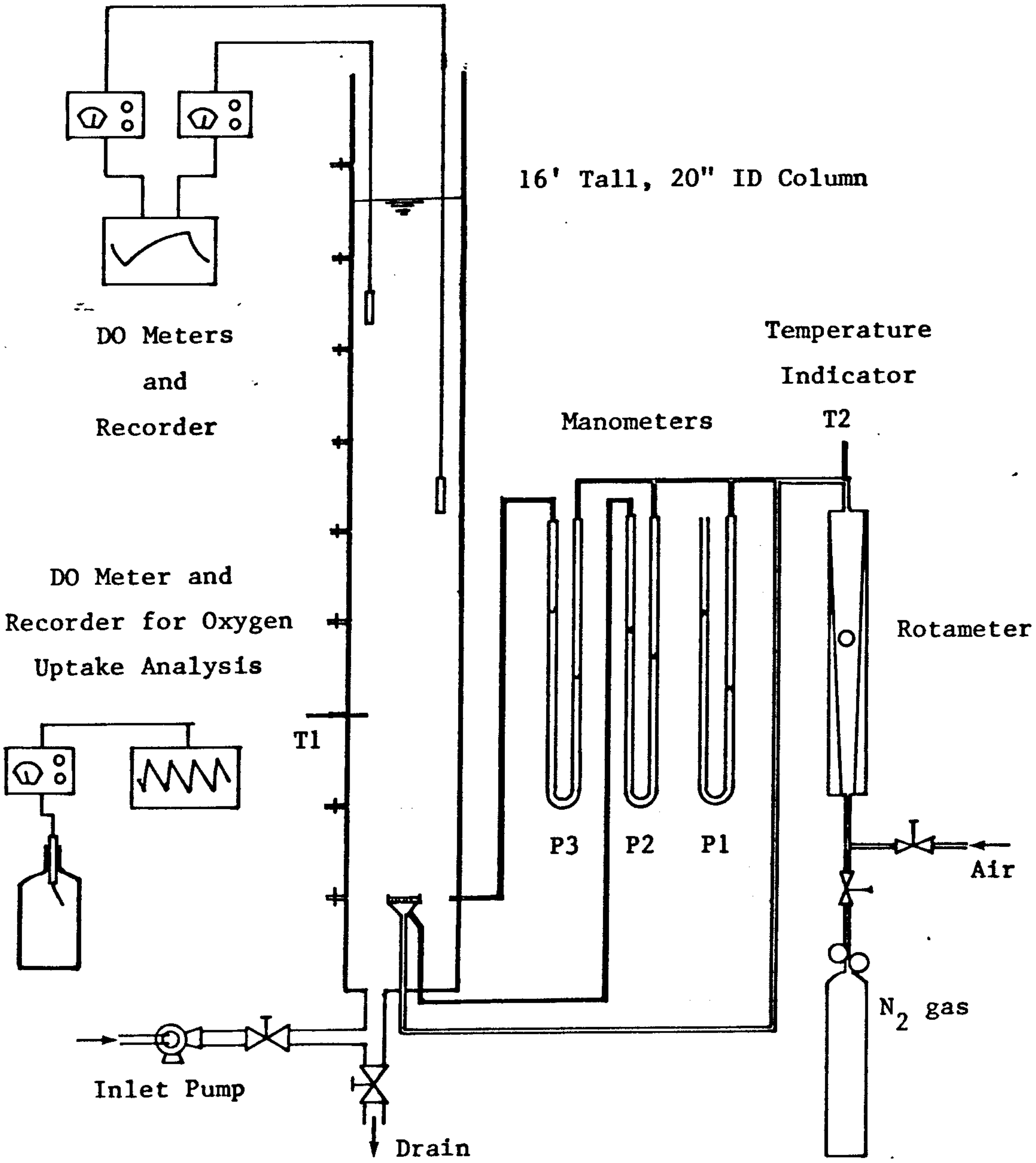


Figure 13. Experimental Set-Up for Diffused Aeration Column

T2 = compressed air temperature immediately following
the rotameter

T3 = ambient temperature

T4 = temperature of inlet air to the main compressors
of the plant

T1 and T2 were monitored using Atkins multi-channel temperature indicator (Model 3H51-F47) and T3 and T4 using Hewlett Packard Voltmeter. Air flow rate was measured by the rotameters and checked with the pressure loss across the diffuser orifice. The measured air flow rate under the process condition was subject to correction for pressure, moisture content, temperature, and gravity. Since the elevation of Los Angeles area is not much higher than sea level, the gravity effect on air flow rate measurement was ignored. The correction factor for pressure and temperature was:

$$f_{pT} = \frac{p_{comp}}{p_{std}} \cdot \frac{T_{std}}{T_{comp}} \qquad (62)$$

where f_{pT} = correction factor for pressure and temperature

p = pressure

T = temperature

comp = subscript for compressed air

std = subscript for standard condition

and the correction for moisture contents was:

$$f_m = \frac{1 - p_v/p}{1 - p_{Bv}/p} \tag{63}$$

where f_m = correction factor for moisture contents
v = subscript for vapor
B = subscript for bubble

ASCE recommends 36% relative humidity at 20°C and 1 atmospheric pressure as a standard condition. Vapor pressure for given temperature was corrected using chemistry handbook (1979). The air flow rate at standard condition was calculated as;

$$G_{s_{std}} = f_{pT}^{2} \cdot f_m \cdot G_s \tag{64}$$

The pressure drop across the orifice in the diffuser unit was converted to the value under standard conditions using suitable correction factors. The air flow rate as a function of pressure drop under standard condition is presented in Figure 14. This correlation was used as a double check for the air flow rate measurement.

Mixed-liquor was taken between 15 and 25 feet from inlet end of aeration tank to ensure complete mixing of primary effluent with return sludge and relatively high uptake rates (40 - 70 mg/l/hr). A maximum of seven experiments were performed with each batch of mixed-liquor.

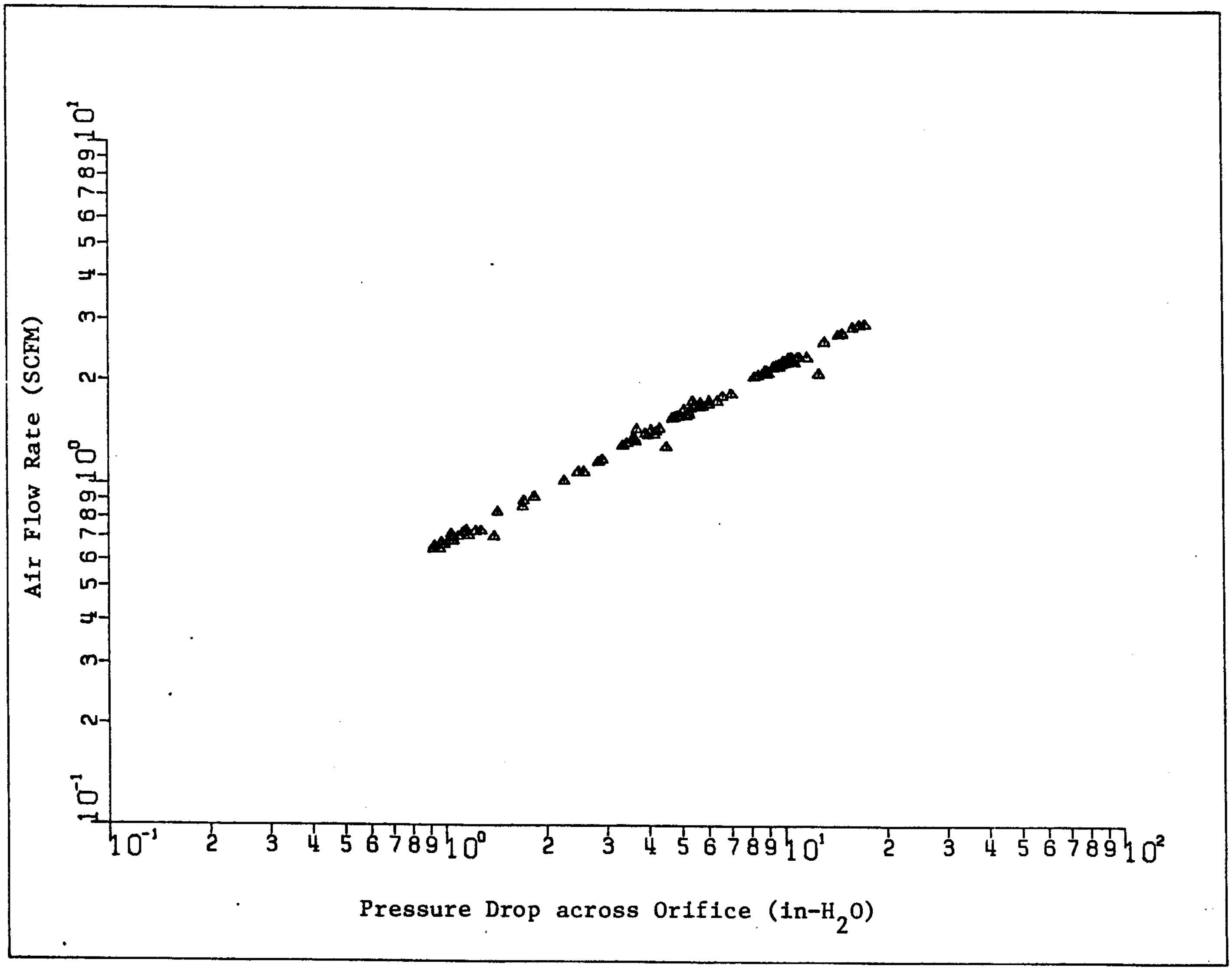


Figure 14. Air Flow Rate vs. Pressure Drop Across Orifice

D. EXPERIMENTAL PROCEDURE

All clean water tests were performed prior to contamination of the equipment by detergents or mixed-liquor. The various water depths studied were 5, 10 and 14.17 ft. (average water depth in aeration basin is 14 ft 2 in). Air flow rate ranged from 0.625 to 2.5 scfm, complying to the diffuser manufacturer's recommended lower limit and the practical upper limit due to air pressure in the main pipe line of the plant. To prevent the diffuser unit from flooding when not performing tests, air or nitrogen gas was continuoulsy supplied at or above the lower limit of air flow rate.

DO probes were calibrated by the Azide modification of Winkler method before the experiments. Dissolved oxygen concentration samples were taken from the column to check any departure from calibration after each experiment.

An anionic detergent, dodecyl sodium sulfate (DSS) of 95% practical grade (Baker), was used as a surfactant. The appropriate amount of DSS to obtain a predetermined concentration was dissolved in a 2-liter beaker, and was poured into the column containing tap water.

The experimental procedure for aeration test on tap water and water containing DSS can be summarized as follows:

(1). Assemble the experimental apparatus: rotameters, manometers, DO meters and probes, recoder, thermocouples and DC volt meter, and nitrogen cylinder.

(2). Calibrate DO probes with Winkler method.

(3). Connect air lines and supply air to diffuser.

(4). Fill the column with tap water to the predetermined test depth.

(5). Submerge DO probes at 2 to 6 feet below the water surface.

(6). Switch gas flow to nitrogen for deoxygenation. In the clean water test with and without surfactants, the dissolved oxygen level was lowered to less than 1 mg/l.

(7). Switch gas flow to air for aeration. Aeration was continued until at least $4/K_La$ time was reached or until DO concentration did not vary more than 0.1 mg/l in 3 - 4 minutes.

(8). After each aeration test, check the calibration of probes with Winkler method and correct any departure from calibration.

(9). Repeat steps 6 through 8 for another air flow rate.

For mixed-liquor, the above procedure was subject to the following modification.

(4). Fill the column with mixed-liquor taken from 15 to 25 ft location in tank 1 to the prescribed test depth.

Take at least 3 uptake rate samples from the aeration tank during filling period, and then take uptake rate samples from the column every 10 to 15 minutes.

(6). Lower the dissolved oxygen concentration close to 1 mg/l, but never less than 1 mg/l, using nitrogen gas.

(7). If the oxygen uptake rate decreased with time during experiment, the steady state value of DO concentration was not obtainable. In this case, the experiment was terminated at the expected value of $4/K_La$.

The deoxygenation of mixed-liquor depended on the rate of microbial oxygen uptake rate in mixed-liquor. When the microbial oxygen uptake rate was lower than the oxygen transfer rate at the lowest available air flow rate, nitrogen gas was used to lower dissolved oxygen concentration. When the microbial oxygen uptake rate was much higher than the available lowest oxygen trasfer rate, experiment was performed and the parameters were evaluated from the decreasing dissolved oxygen concentration versus time data. For this convex-down experiment, care must be taken so that dissolved oxygen must not be lowered below the critical concentration. In an either concave-up or convex-down experiment, the dissolved oxygen concentration variation during the experiment should be large enough to minimize the effect of measuring error of time and DO concentration. For this research, the dissolved oxygen concentration variation was chosen to be at least 4 mg/l.

IV. SURFACE AERATION

A. DERIVATION OF NONDIMENSIONAL PARAMETERS

Either in cylindrical or square shape aeration vessels equipped with baffles which prevent vortexing, the essential geometric and dynamic similarity required for oxygen transfer is around the rotors assuming the liquid in the entire vessel is mixed. This assumption will not require the similarity of the shape and size of the aeration vessels. Therefore the following variables are required to described fluid motion and oxygen transfer;

$$K_L a_{20} \cdot V = f\ (D, DI, S, HI, D_L, \rho, \nu, \sigma, n, g, P) \qquad (65)$$

The variable $(K_L a_{20} \cdot V)$ is the standard oxygen transfer rate per unit dissolved oxygen concentration deficit. From these twelve variables, nine nondimensional parameters can be derived as:

$$Y_H = f\ \left(\frac{DI}{D}, \frac{S}{D}, \frac{HI}{D}, \text{Re}, \text{Fr}, \text{Sc}, \text{Ne}, \text{We} \right) \qquad (66)$$

where Y_H = Oxygen transfer number

$$= \frac{K_L a_{20} \cdot V}{DI^5} \cdot \left(\frac{\nu^5}{g^4} \right)^{1/3} \qquad (67)$$

For geometric similarity, first three nondimensional parameters can be kept constant. If the Reynolds number is greater than 10^5, Re is independent of power input and oxygen transfer rate (Rushton et al., 1950a, 1950b).

The surface active agents at low concentration do not alter the bulk liquid properties significantly, such as diffusion coefficient and viscosity. Plevan and Quinne (1966) studied the gas transfer to the quisient surfactant layer and found that absorption of carbon dioxide to pure water deviated considerably from pure diffusion process due to convection, but absorption to 0.5 % CMC gel solution, in which convection was restricted, followed closely with the desorption data from pure water. This indicates no significant change of diffusion coefficient in the surfactant solution. Nguyen et al. (1972) analyzed the experimental data of Whitaker and Pigford (1966) of sulfur dioxide absorption to creosol green solution and found that diffusion coefficient to quisient surfactant film was 3 orders of magnitude less than diffusion coefficients in bulk liquid. However they found that diffusion coefficient in bulk liquid increased with temperature but was not affected by the concentration to remain the value of pure water. The presence of a surfactant of 10^{-5} mole/l does not change liquid viscosity. Therefore, the variation of Schmidt number, if any, can be ignorded, and the oxygen transfer number Y_H is the function of Froud, Weber and Power numbers only.

$$Y_H = f\ (Fr,\ We,\ Ne) \tag{68}$$

B. EXPERIMENTAL SEP-UP FOR SURFACE AERATION

Experimental work for the dimensional analysis of surface aeration was conducted at the Water Quality Control Laboratory at UCLA. Three cylindrical aeration tanks of laboratory scale were employed and their dimensions are listed in Table 3.

Each aeration tank was equipped with four baffles with a width equal to one-tenth of the tank diameter and from the bottom of the tank to several inches above water surface. The impellers were the three-blade propellers in two sizes of 2 and 3.5 inches in diameter. The driven system consisted of two parts, a permanent magnetic DC motor-generator (Motomatic by Elector-Craft Co.) a solid state electronic controller (Master servodyne by Cole Parmer). The motor provides an output torque to drive the impeller and generator sends a feedback signal to the controller, which compares this signal to an internal reference value and adjusts the current supply in order to maintain the rotational speed of the impeller. This additional electrical current is proportional to the torque imposed on the impeller by the liquid, and is indicated on a millivolt meter. The millivolt reading can be converted to torque using the calibration chart supplied by the manufacture (Figure 16).

Dissolved oxygen concentration in the test liquid was continuously monitored by the two YSI DO probes and meters and recorded on a strip chart recorder during experiments. The schematic diagram of the set-up is presented in Figure 17.

Aeration Tank			Water	
volume (gal)	Diameter (inch)	hight (inch)	depth (inch)	volume (gal)
30	18	30	22.7	20
55	22	36	27.3	45
200	36	48	43.1	190

Table 3. Dimensions of Laboratory-Scale Aeration Tanks

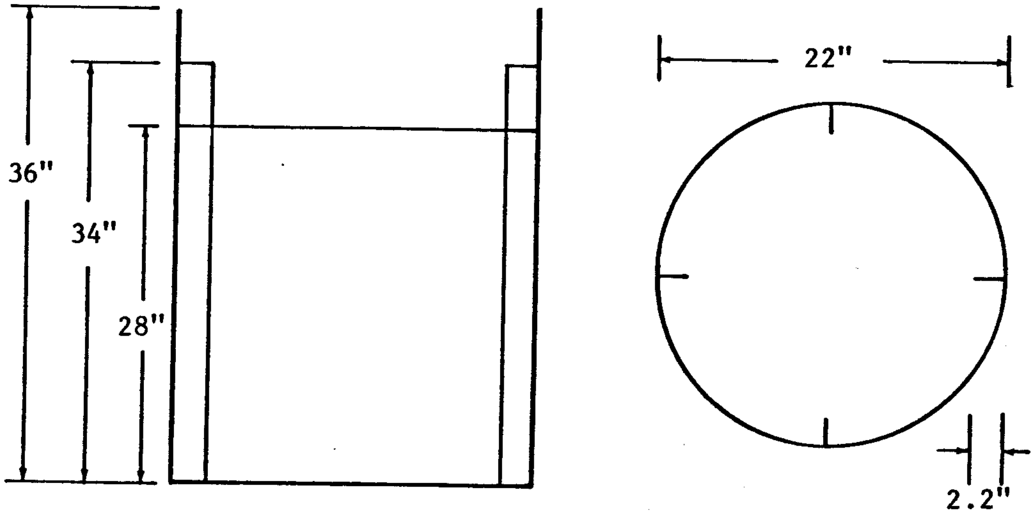
The power consumption for agitation was calculated using the following equation:

$$HP = \frac{dW}{dt} \tag{69}$$

$$= L \cdot n \cdot \frac{1}{12} \cdot \frac{1}{60} \cdot \frac{1}{550} \tag{70}$$

where HP = impeller horse power (hp)
W = work done by rotors
L = torque inposed on impeller (in-lbs)
n = rotational speed (RPM)

To study the effect of surface active agent on the surface aeration performance, dodecyl sodium sulfate was used in the concentration range from zero to 15 mg/l.



Top View

Side View

Figure 15. Geometry of 55-gal Aeration Tank

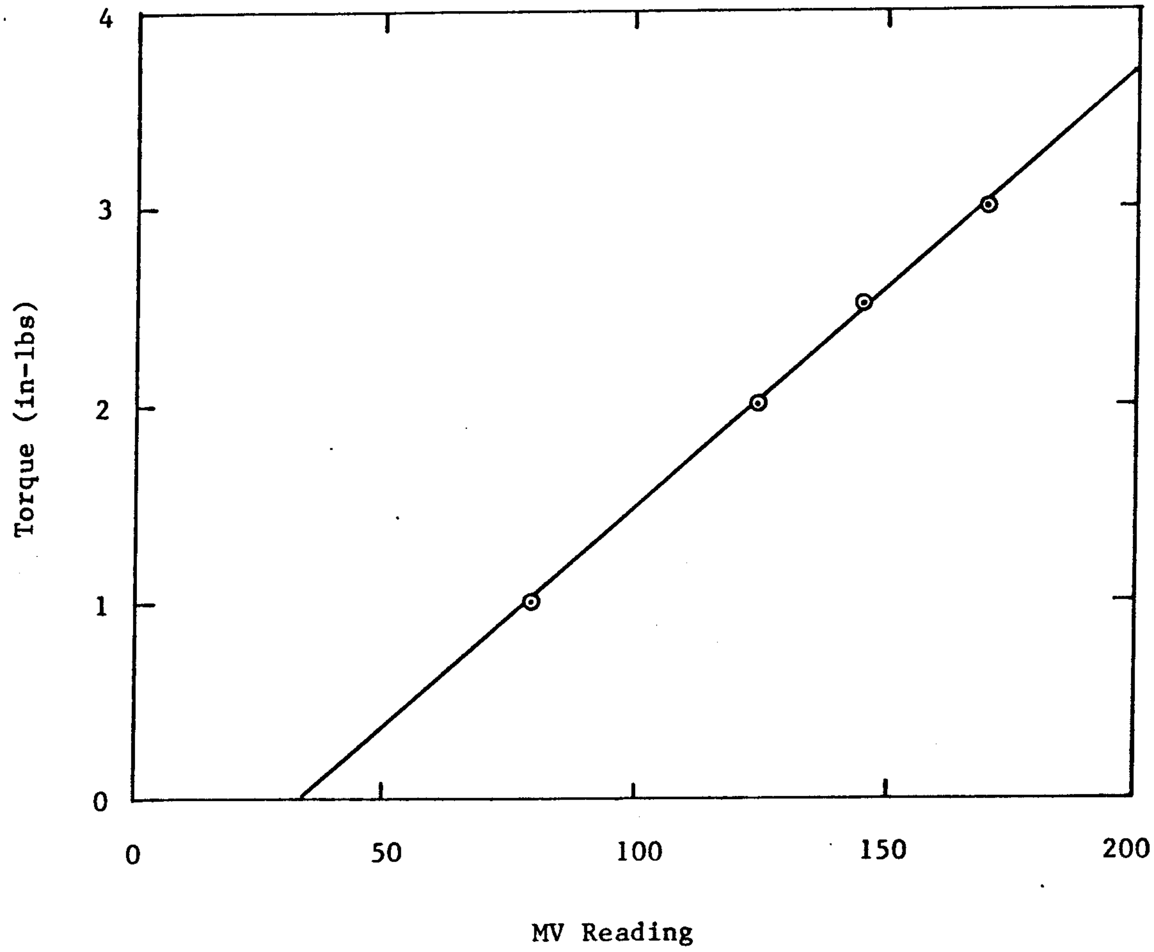


Figure 16. Calibration Chart for Electronic Controller

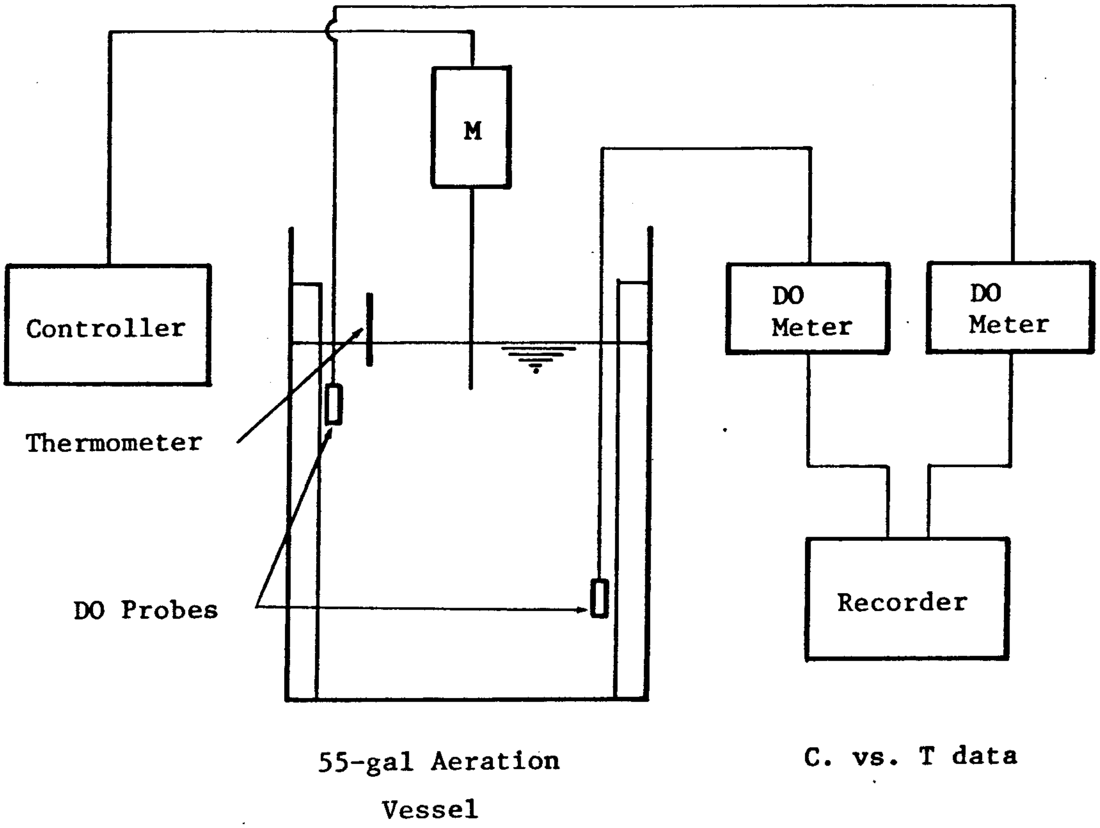


Figure 17. Schematic Diagram of Experimental Set-Up for Surface Aeration

C. EXPERIMENTAL PROCEDURE

(1). Wash the aeration vessel thoroughly with tap water.

(2). Assemble the Experimental apparatus and fill the vessel with tap water.

(3). Calibrate DO probes with Winkler method and submerge them in the vessel.

(4). For the test of surfactant, dissolve a proper amount of surfactant in a 500 ml beaker and mix the concentrate in the aeration vessel.

(5). Add concentrated cobalt chloride solution to the water in the vessel to make 0.5 mg/l as cobalt ion.

(6). Dissolve sodium sulfite in 300 ml of tap water and mix into the water in the vessel. One and one-fourth to one and one-half the stochiometric requirement of sodium sulfite was used.

(7). Mix water in the vessel until all oxygen probes submerged at different location become stabilized at zero mg/l dissolved oxygen concentration.

(8). Start aeration and monitor the dissolved oxygen concentration versus time data until time at leat $4/K_La$.

(9). Take samples and analyze dissolved oxygen concentration with Winkler method.

(10). Repeat steps 6 through 10 for different power input.

V. DATA ANALYSIS

A. PARAMETER ESTIMATION

For the nonsteady state reaeration test in the clean water, dissolved oxygen concentration versus time data can be described with equation 37 for oxygen mass balance in the liquid. Parameter estimation techniques for this test are differential technique, log-deficit technique, and exponential technique. The exponential technique has advantages over other techniques, such as no requirement for data truncation. The details on these models can be found in the literature (Stenstrom, 1979; Baillod and Brown, 1983) and are not discussed here. However, for the exponential model of equation 38, two well known algorithms are applicable, Box method and linearized model by Taylor series (Stenstrom et al., 1981) The linearized model was recommended by ASCE Subcommettee for Oxygen Transfer Standardization. Since the linearized model algorithm has been proved effective where good starting points of the unknown parameters are available, it has been used in this research. The Box method has been also effective for clean water test data analysis (Stenstrom, 1979; Hwang and Stenstrom, 1979). For the clean water data anlysis, the linearized model converges faster, but it requires derivatives with respect to unknown variables, and it is not applicable to data from nonsteady state dirty water tests with changing K_La value. For these data, the Box method has been used since it does not require derivaties.

B. CLEAN WATER TEST DATA

To estimate the volumetric mass transfer coefficient and saturation dissolved oxygen concentration from clean water test data, the linearized model is applicable. Its algorithm is briefly presented in the following discussion.

The basic equation for the parameter estimation using linearized exponential model is equation 38. The Taylor series expansion of the equation about the parameters becomes:

$$C_{est}^{n+1} = C_{est}^{\ n} + \frac{\partial C}{\partial K_L a}\bigg|^{n} \cdot \Delta K_L a^n + \frac{\partial C}{\partial C_\infty^*}\bigg|^{n} \cdot \Delta C_\infty^{*n} + \frac{\partial C}{\partial C_0}\bigg|^{n} \cdot \Delta C_0^{\ n} \qquad (71)$$

$$\text{and} \quad \frac{\partial C}{\partial K_L a} = t \cdot (C_\infty^* - C_0) \cdot \exp(-K_L a \cdot t) \qquad (72)$$

$$\frac{\partial C}{\partial C_\infty^*} = 1 - \exp(-K_L a \cdot t) \qquad (73)$$

$$\frac{\partial C}{\partial C_0} = \exp(-K_L a \cdot t) \qquad (74)$$

Then the least squares objective function is,

$$S_L = \underset{\Delta K_L a,\ \Delta C_\infty^*,\ \Delta C_0}{\mathrm{MIN}} \sum (C_{obs} - C_{est}^{n+1})^2 \qquad (75)$$

After the evaluation of three variables in equation 75, the new set of parameters are calculated as follows:

$$K_La^{n+1} = K_La^n + \Delta K_La^n \tag{76}$$

$$C_\infty^{*n+1} = C_\infty^{*n} + \Delta C_\infty^{*n} \tag{77}$$

$$C_0^{n+1} = C_0^n + \Delta C_0^n \tag{78}$$

With these new estimates, the algorithm repeats until the convergence criteria are satisfied. This algorithm is fast enough to converge within several iterations with reasonable starting point. More details can be found else where (Stenstrom et al., 1981; Baillod and Brown, 1983). Based upon this algoritm, a computer code was written.

This program is applicable to detergent water data, and also to oxygen uptake rate data of equation 47.

C. MIXED LIQUOR TEST DATA

When both the oxygen uptake rate and volumetric mass transfer coefficient are constant, the mass balance equation 40 describes the concentration varation. The linearized technique can be used to estimated parameters, K_La, C_R, and C_0 from equation 41, and then saturation concentration can be evaluated from equation 42.

If the oxygen uptake rate changes exponentially with time, and volumetric mass transfer coefficient can be assumed constant, the dissolved oxygen concentration is decribed by equation 48. As mentioned earlier, variables, Ku, R_0 and R_c can be determined separately from dissolved oxygen concentration data as long as dissolved oxygen level is kept higher than critical value. To apply linearized model for this equation, the new set of derivatives is required.

$$\frac{\partial C}{\partial K_L a} = G_4 + G_2 \cdot \exp(-Ku \cdot t) - (G_4 + G_2) \cdot \exp(-K_L a \cdot t) + t \cdot (C^*_\infty - G_3 - G_1 - C_0) \cdot \exp(-K_L a \cdot t) \quad . \quad (79)$$

where

$$G_1 = \frac{R_0}{K_L a - Ku} \quad (80)$$

$$G_2 = \frac{R_0}{(K_L a - Ku)^2} \quad (81)$$

$$G_3 = \frac{R_c}{K_L a} \quad (82)$$

$$G_4 = \frac{R_c}{K_L a^2} \tag{83}$$

and $\frac{\partial C}{\partial C_\infty^*}$ and $\frac{\partial C}{\partial C_0}$ are defined by equations 69 and 70.

With this set of derivatives, linearized model has been used successfully.

When the oxygen uptake decreases rapidly and volumetric mass transfer coefficient is dependent on its value, equation 56 is suitable. The C value can be obtained using the fourth order Runge-Kutta numerical integration method. An IBM applications program "Continuous systems modeling program III" (CSMP III) (IBM,1972) can be used to integrate these equations. The Box algorithm has been used to estimate parameters with least squares objective function. The variables to be subjected to the minimization in the Box algorithm are not the incremental steps of parameters, as in linearized model, but the parameters themselves, as follow:

$$S_B = \min_{K_L a,\ C_\infty^*,\ C_0} \sum (C_{obs} - C_{est}^{n+1})^2 \tag{84}$$

The algorithm can be summarized as follows:

(1). Select 4 sets of parameter estimates and evaluate the error for each set of estimates using the fourth order Runge-Kutta integration method.

(2). Step size = 1.3.

(3). Determine the worst set of parameters.

(4). Calculate centroid of the remaining sets.

(5). Project the new set of estimates.

(6). Evaluate error for new set.

(7). If necessary, half the step size.

(8). Repeat step 2 through 7 until finish criteria are satisfied.

Computer code for this algorithm was developed previously (Stenstrom, 1971) for general purposes and was modified for this research.

For the continuous flow system, if oxygen uptake rate and K_La value can be assumed invariant, the linearized model can be used for equation 59 to estimated K_La', C_R', and C_0, then K_La, C_∞^*, can be evaluated from equations 60 and 61.

VI. EXPERIMENTAL RESULTS

A. DIFFUSED AERATION

A-1. CLEAN WATER TEST

The proper determination of alpha and beta depends on the accurate measurements of volumetric mass transfer coefficient and saturation dissolved oxygen concentration in clean water. Since the small quantity of trace contaminants has great influence on volumetric mass transfer coefficient, the experimental column was carefully cleaned and tap water tests were performed prior to surfactant water or mixed-liquor tests.

The range of experiments covered 14'2", 10' and 5' of water depth, from 0.6 to 2.8 scfm of air flow rate. The water temperature varied from 25 to 30 ^{o}c. Clean water test result is presented in Table 4. Volumetric mass transfer coefficient under process condition was converted to that at standard condition using theta value of 1.024. The volumetric mass transfer coefficient versus air flow rate is plotted in Figure 18 for each water depth.

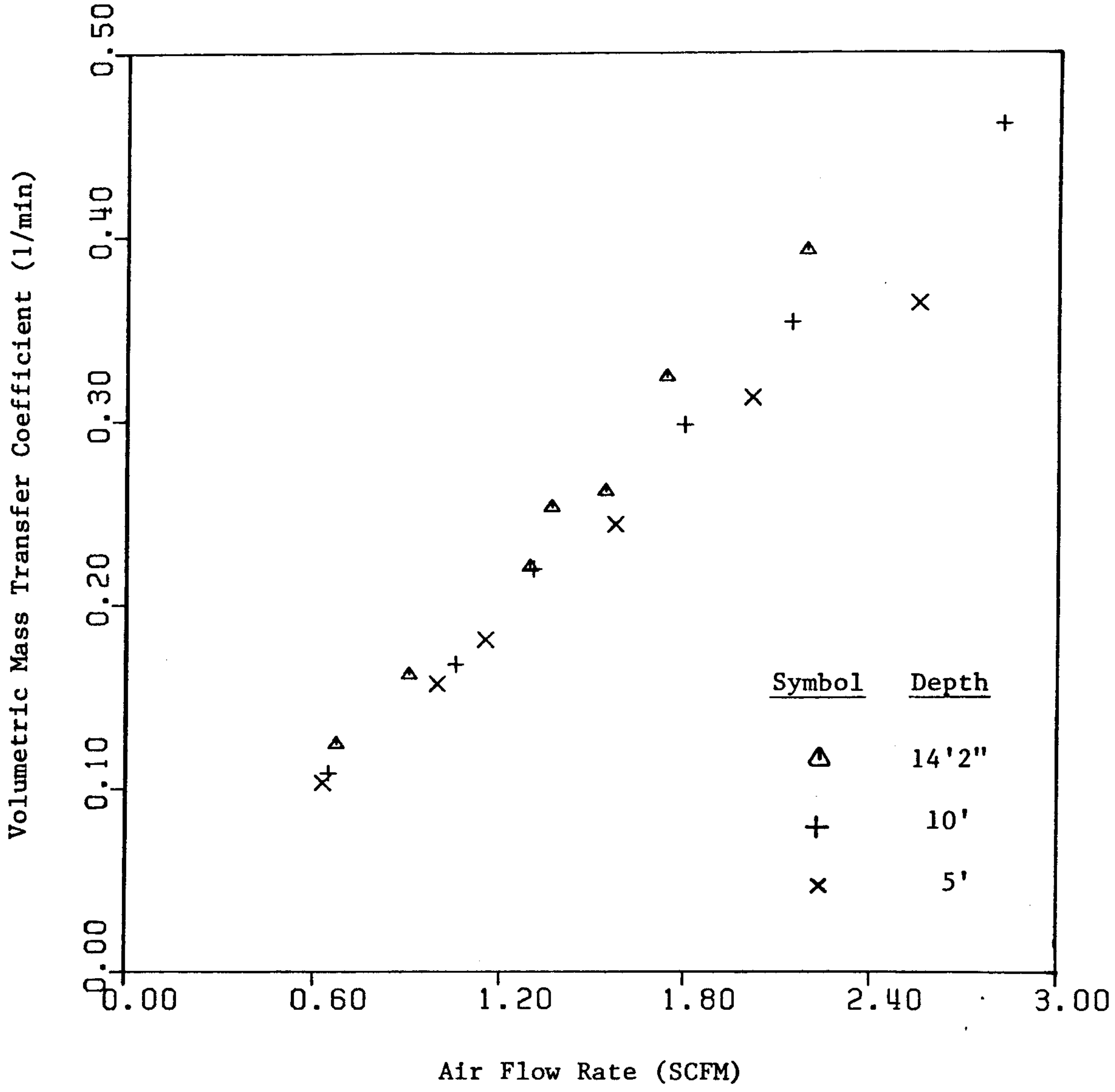


Figure 18. K_La vs. Air Flow Rate in Tap Water

Effect of Air Flow Rate

The data show a linear dependence of volumetric mass transfer coefficient on air flow rate at each water depth. This linear dependency of volumetric mass transfer coefficient on the gas flow rate has been reported by other investigators as disscused in section II-D.

Figure 18 shows K_La value increases with liquid depth increases from 5 ft to 14 ft 2 in, and then decreases at 15 ft. This trend does not follow other researchers' findings. Jackson and Shen (1978) reported that k_La changes with $(-0.45 \sim 0.55)$ power of liquid depth. Yunt (1982), Bacon et al. (1977), and Urza and Jackson (1975). also reported slight decreases of K_La with liquid depth. However, in all those experiments, diffuser units were located at the bottom or at least close to bottom. In this research, diffuser unit was lifted 2 ft above the column bottom, and the extra depth (as high as 40 % at 5 ft water depth) had more effect especially at lower water depth. Consequently, overall K_La value decreased at lower water depth.

Saturation Dissolved Oxygen Concentration

Figure 19 shows the saturation dissolved oxygen concentration in three water depths. There is no apparent variation of saturation concentration with air flow rate, but increased with water depth, as shown in Figure 20. The extrapolted value at zero water depth of 9.06 mg/l is close to surface saturation concentration 9.07 at 20°c.

water depth (ft)	air flow rate (scfm)	K_La (1/min)	C^*_∞ (mg/l)
14.8	2.62	0.378	10.2
	1.66	0.286	10.2
	1.07	0.187	10.2
	1.35	0.232	10.1
	0.71	0.119	10.4
14.1	2.19	0.394	9.9
	1.74	0.324	9.8
	1.36	0.253	9.8
	0.67	0.124	9.8
	1.54	0.262	10.1
	1.29	0.221	10.1
	0.90	0.162	9.9
9.9	2.82	0.463	9.8
	2.14	0.354	9.9
	1.80	0.298	9.8
	1.31	0.219	9.8
	1.06	0.167	9.7
	0.65	0.109	9.7
5.0	2.55	0.365	9.2
	2.02	0.313	9.3
	1.57	0.244	9.3
	1.15	0.181	9.2
	1.00	0.157	9.2
	0.63	0.104	9.2

Table 4. Summary of Tap Water Test Result in Aeration Column

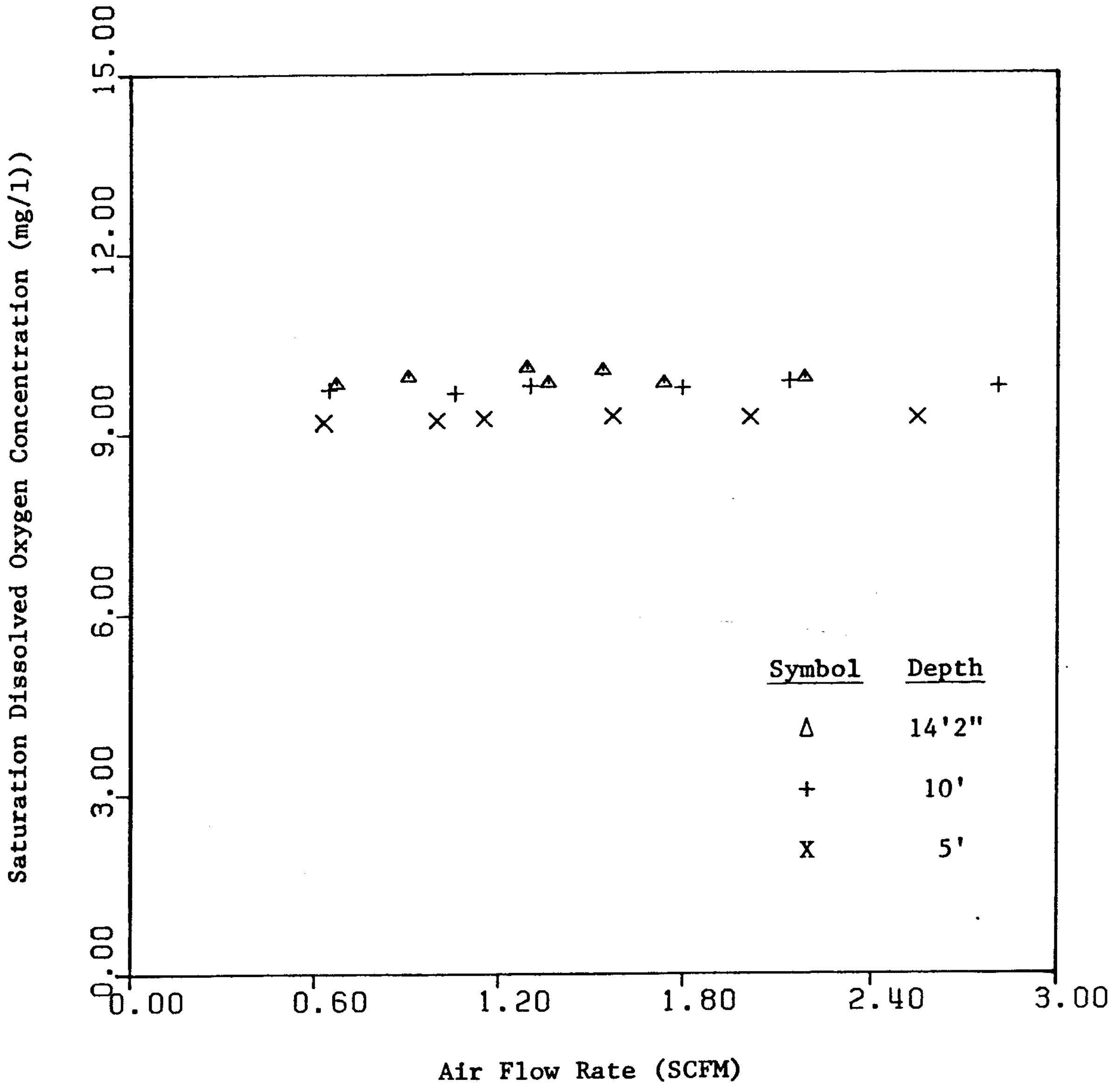


Figure 19. Saturation Dissolved Oxygen Concentration in Tap Water

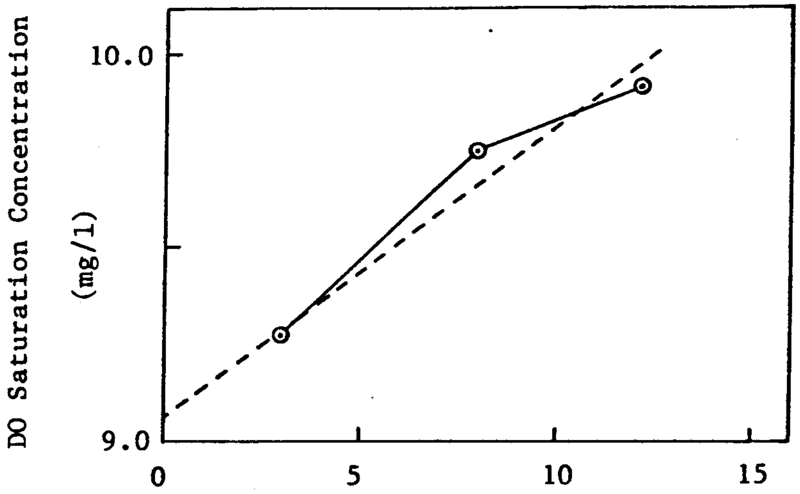


Figure 20. Saturation DO Concentration vs.
Diffuser Submergence

A-2. EFFECT OF SURFACTANT ON OXYGEN TRANSFER

The summary of experimental results is presented in Table 5. As seen in the experiment number 32 to 39, the replication of volumetric mass transfer rate is excellent, considering experimental difficulties associated with surfactant, with standard error of 3.5 % from mean value in each batch of test liquid. However, the results from batch to batch differ considerably. In the first two days, dodecyl sodium sulfate from Mallinkrodt Inc. was used. This chemical was stripped easily from liquid with rising air bubbles to condense at the surface, so that surface tension rapidly recovered to the value of tap water. In the rest of experiments, DSS from J.T. Baker was used.

The variation may be partially due to mixing technique of concentrated solution with main water body. In the early series of experiments of DSS solution, the 2 liter concentrate solution was mixed to water from the top of the column. This concentrated solution of 2.19 g/l was close to the critical micelle concentration, which is 2.48 g/l at 40°C (Rosen, 1978). Consequently, the surfactant molecules of the high concentration in the top section of the column might exceed the critical micelle concentration and the micelles were rejected before they were mixed with water in the lower portion of the column. The surfactant molecules once stripped and condensed at the surface stayed on the top of foam and was not mixed again to the water. In the later series, the concentrated solution was continuously added slowly as the water depth increased and the foams were continuously broken down with spray of water from the top, so that it was possible to achieve relatively uniform distribution of detergent. This means the difference

Experiment Number	Date	Water Depth (ft)	Air Flow Rate (SCFM)	K_La (1/min)	C^*_∞ (mg/l)
32	9/15	14.3	1.66	0.139	10.4
33		14.3	1.67	0.132	10.6
34		14.3	1.68	0.142	10.5
35	9/16	14.3	2.22	0.208	10.1
36		14.3	2.22	0.229	9.8
37	9/17	14.3	2.23	0.286	10.1
38		14.3	2.24	0.292	9.8
39		14.3	2.24	0.277	9.9
40	9/22	14.2	2.23	0.152	10.1
41		14.2	1.38	0.103	9.8
42		14.2	0.69	0.050	9.5
43	9/23	10.0	2.14	0.125	9.3
44		10.0	1.34	0.095	9.3
45		10.0	0.66	0.051	9.1
46	9/24	5.0	2.08	0.128	8.7
47		5.0	1.26	0.094	8.8
132	5/6	14.3	2.20	2.244	10.2
133		14.4	1.71	0.180	10.0
134		14.3	0.90	0.113	9.7
135	5/6	10.0	2.16	0.263	10.4
136		10.0	1.62	0.225	9.2
137	5/7	10.0	0.89	0.130	9.4
139	5/7	5.0	2.09	0.21	9.2
139		5.0	1.54	0.181	9.1
140		5.0	0.82	0.113	8.8

Table 5. Summary of Detergent Test Result in Aeration Column

of chemicals from those two manufacturers was not entirely responsible for variation.

The variation of alpha factor with air flow rate is shown in the data, i.e., alpha value in surfactant solution was higher at lower air flow rate. The lowest air flow rate in each batch was tested last, but the result of Run 32 through 39 gave no distinctive time effect on alpha factor.

Volumetric mass transfer coefficient or alpha value of Run 132 to 140 was much higher than the earlier test result. Those later experiments were conducted after mixed-liquor test. After the mixed-liquor test, the residue on the wall of PVC column was not easy to remove and some may have remained. The surfactant molecules in the later tests might be attached to the residue on the wall, so that they did not have much effect on air liquid interface to lower mass transfer coefficient.

A-3. RESULT OF MIXED LIQUOR TEST

For the mixed-liquor test in the column, three experimental techniques were employed:

(1). Constant OUR nonsteady state batch test

(2). Decaying OUR nonsteady state batch test

(3). Constant OUR nonsteady state continuous flow test

Constant OUR Nonsteady state Batch Test

As shown in preliminary study in the aeration tank, the initial high oxygen uptake rate decreased rapidly to assume a relatively constant value in about one and a half hours. Aeration column was filled at slow speed around 2 hour period for 14 ft 2 in liquid depth. At the beginning of pumping, the mixed-liquor depth was increased rapidly due to hydrualic head from aeration tank, and the rising speed of liquid depth was slowed down as the liquid depth increased. During this pumping period, air flow rate was kept high, so that dissolved oxygen concentration in the liquid column was at or above 3 mg/l. The uptake rate versus time data is given in Figure 21.

In this series of experiments, microbial oxygen uptake rate did not change more than 10 % during each test. The volumetric mass transfer coefficients as a function of air flow rate in mixed-liquor is presented in Figure 22. Unlike tap water, K_La values in mixed-liquor shows slightly higher value in lower water depth.

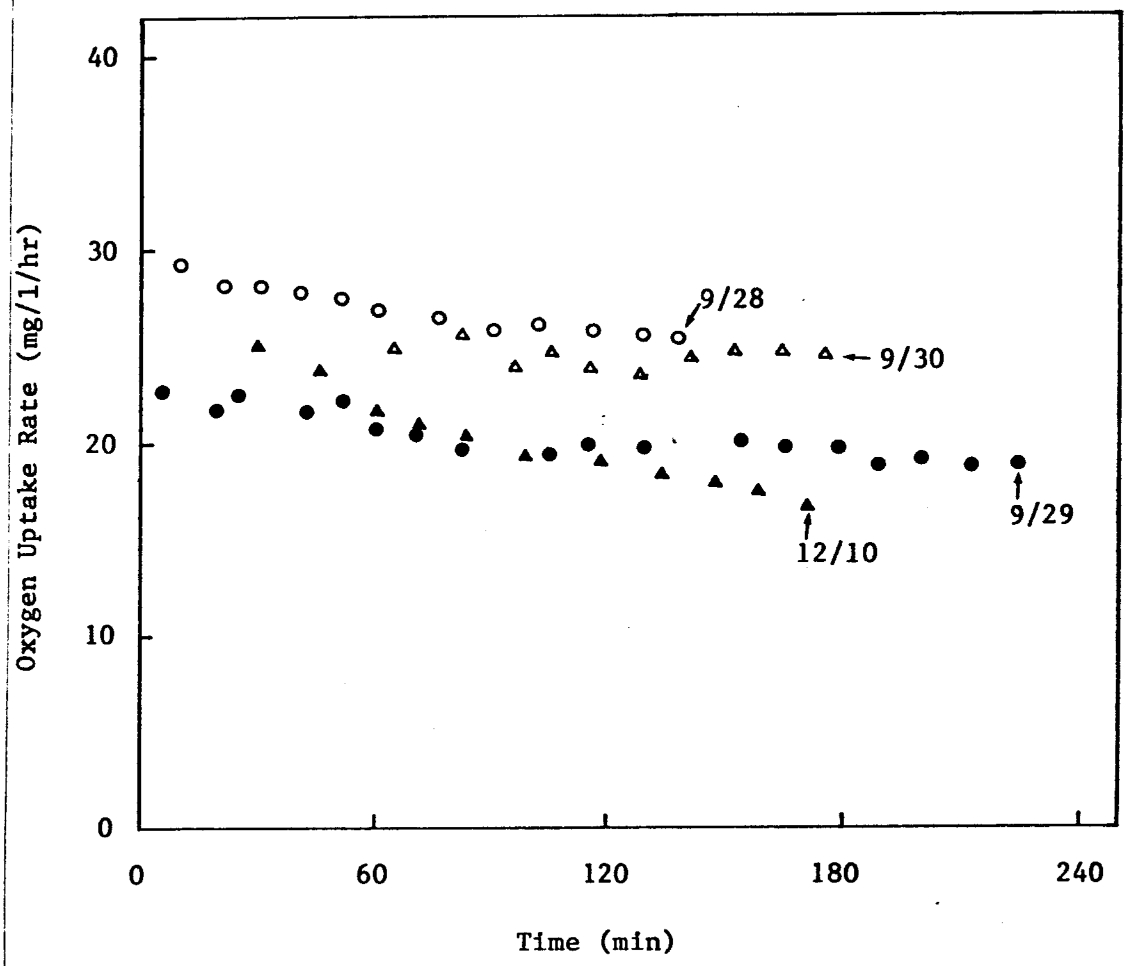


Figure 21. Oxygen Uptake Rates during Constant OUR Tests

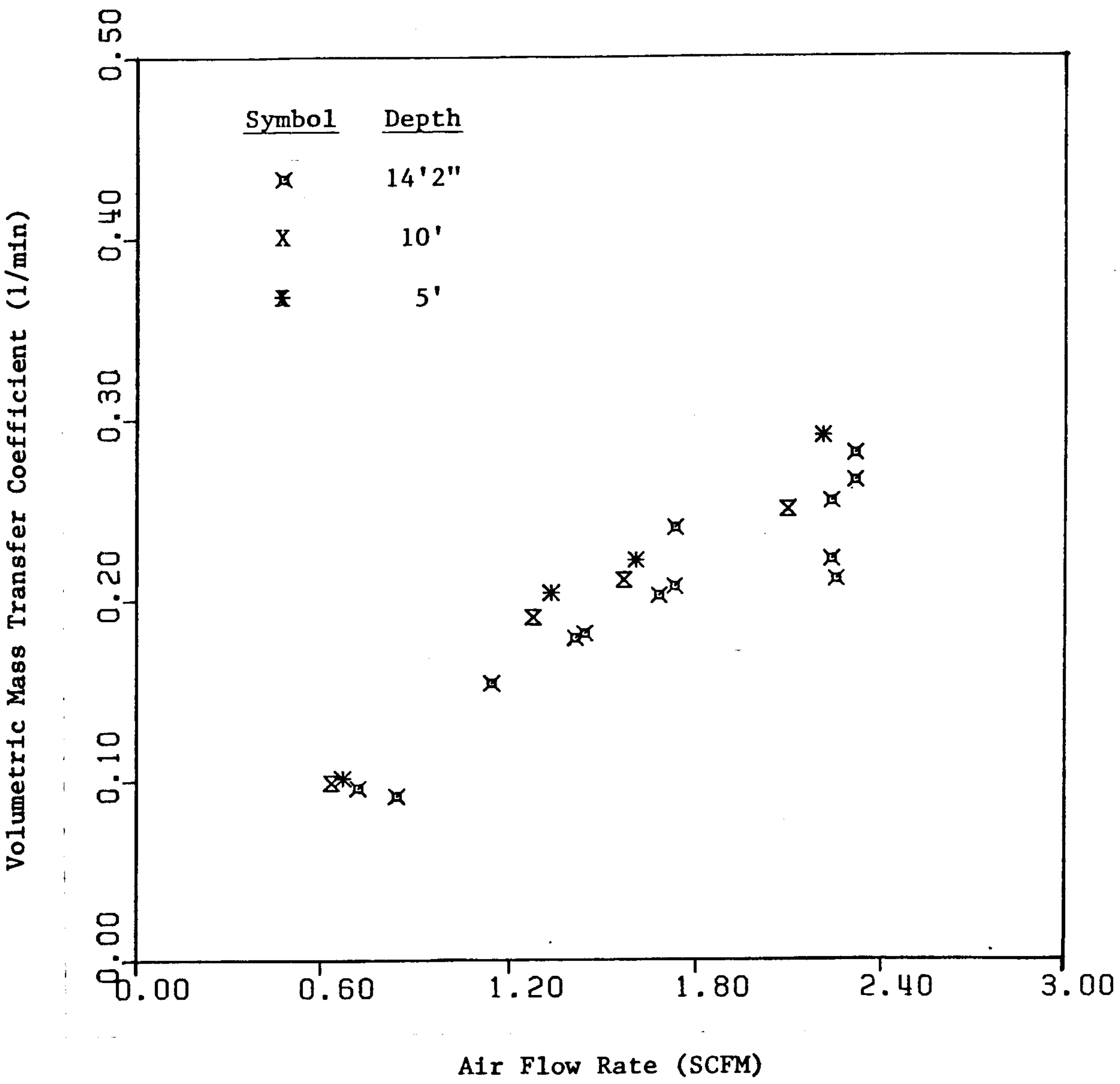


Figure 22. K_La vs. Air Flow Rate in Mixed Liquor in Aeration Column

Decaying OUR Nonsteady State Batch Test

In these experiments, mixed-liquor was taken from 15-foot location in tank 1 to ensure high initial uptake rate and the 14'2" liquid column was filled in about 7 minutes. Figure 7 shows typical response of oxygen uptake rate in rapid filled aeration column. Uptake rate is in the range of 60 to 70 mg/l/hr at the sampling point in aeration tank and that value decreases exponentially to 18 to 22 mg/l/hr in about 2 hours.

Volumetric mass transfer coefficients versus microbial oxygen uptake rates are shown in Figure 24 at 14'2" mixed-liquor depth. The average air flow rate of these nine experiments was 2.294 SCFM with a standard deviation of 0.043 SCFM. Except for one experiment, all results fall in one line showing a linear dependency of volumetric mass transfer coefficient on microbial oxygen uptake rate.

The experimental range for rapidly decaying oxygen uptake rate covered all three liquid depths and at least three air flow rates in each liquid depth and from 20 to 65 mg/l/hr of oxygen uptake rates. The results will be discussed in detail in the next section with the results of other experiment methods.

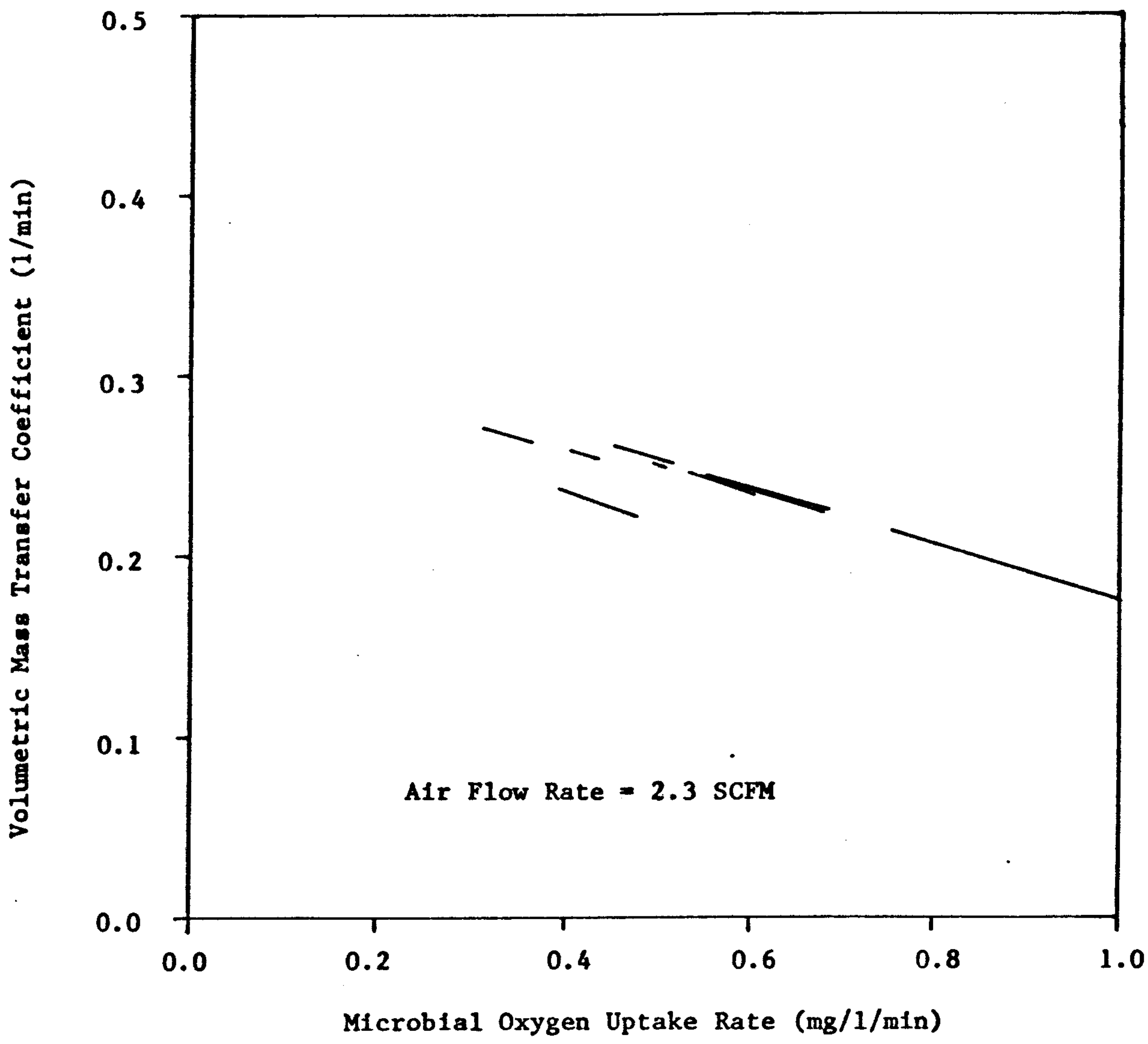


Figure 23. K_La vs. Oxygen Uptake Rate in 14'2" in Decaying OUR Tests

Constant OUR Nonsteady State Continuous Flow Test

The purpose of this experiment was to check the variation of volumetric mass transfer coefficient at higher uptake rate obtained with nonsteady state batch test. At lower uptake rate of endogenous phase, the experimental result from steady state method should be identical to the result from nonsteady state batch test method. Two experiments were conducted for continuous flow mode.

In this series of experiments, after the aeration column was filled with a new batch of mixed-liquor (in about 7 minutes), mixed-liquor was continuously delivered from tank to aeration column to keep high oxygen transfer rate. The summary of experimental results is presented in Table 6.

Discussion of Experimental Results in Mixed Liquor

Figures 24 through 26 show the experimental results in mixed-liquor at three different liquid depths of 14 ft 2 in, 10 ft and 5 ft. Solid lines on the figures indicate the results of nonsteady state tests with changing volumetric mass transfer rate with oxygen uptake rate during each experiment. The length of the line of nonsteady state tests implies the length of time for that test as well as the change of microbial oxygen uptake rate during test. Figure 27 shows variation of alpha value in 14'2" mixed-liquor depth with air flow rates. As the air flow rate per diffuser increased alpha value decreased. At 24 mg/l/hr of oxygen uptake rate, alpha value changed from 0.73 to 0.64 as air flow rate increased from 0.72 to 2.28 scfm. Alpha value was also affected by

water flow rate (gal/min)	water volume (gal)	air flow rate (SCFM)	uptake rate (mg/l/hr)	K_La (a/min)	C^*_∞ (mg/l)
14.0	230	2.24	41.0	0.222	9.6
14.0	230	2.25	39.5	0.211	9.8

Table 6. Summary of Constant Oxygen Uptake Rate, Nonsteady State, Continuous Flow Test.

liquid depth. As mixed-liquor depth increased, alpha value decreased.

A-4. COMPARISON WITH THE RESULT IN FULL SCALE SYSTEMS

The same type of diffuser as the one in this research was used by Yunt in LACSD in two separated studies of full scale systems (Yunt, 1982 and 1983). The first set of data presented in Table 7 is from one of Yunt's experiments using tap water in grid 1 of tank 1 at Whittier Narrows WRP. Water depth was 14.84 feet and the diffusers were mounted on header 2-foot off the bottom. The total number of 724 diffusers were laid in 30' x 100' floor as seen in Figure 11 to give 4.1 sq ft per unit diffuser. The second set of data were from the rectangular tank of 20' x 20' x 25' in LACSD's Joint Water Pollution Control Plant in Carson, California. Total 98 diffusers were installed to give same floor area per unit diffuser as above. Total pressure and static heads are also provided in the Table 7.

These two sets of data are plotted in Figure 29 as K_La versus superficial air flow rate. In both cases, K_La values are about 20 % higher than the values from the aeration column. This difference is believed due to wall effect which changes the mixing pattern, bubble rising velocity, and bubble entrapment.

Mixed-liquor tests in the aeration tank 1 in Whittier Narrows WRP were conducted by Mueller (1982) in summer of 1981. The data were analyzed using Dual Nonsteady State model and Steady State model. These tests were performed in grid 2 and 3 where floor area per unit diffuser are 5.05 and 8.52 sq ft, respectively. The summary of experimental

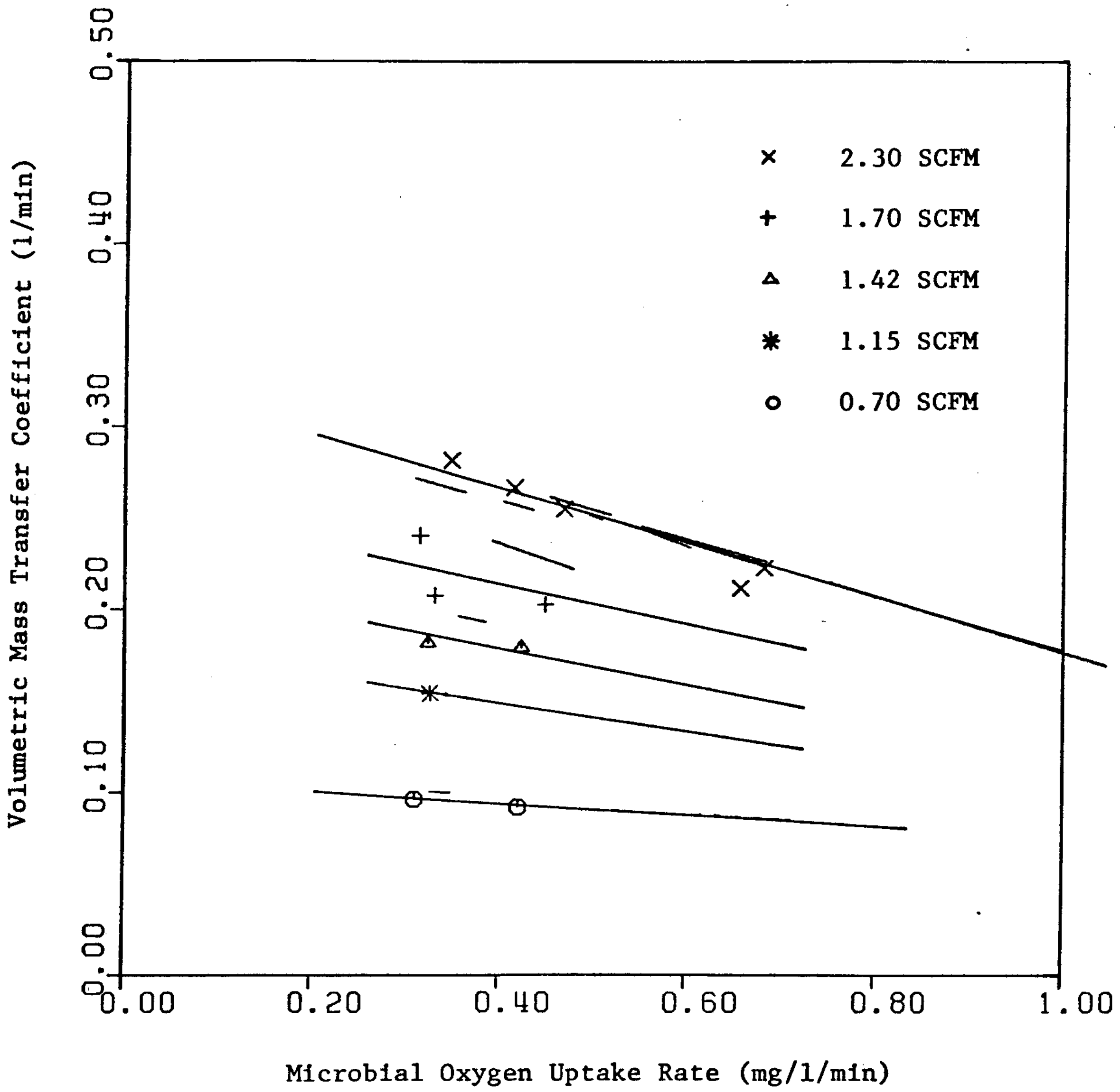


Figure 24. K_La vs. OUR at various Air Flow Rate in 14'2" Mixed Liquor

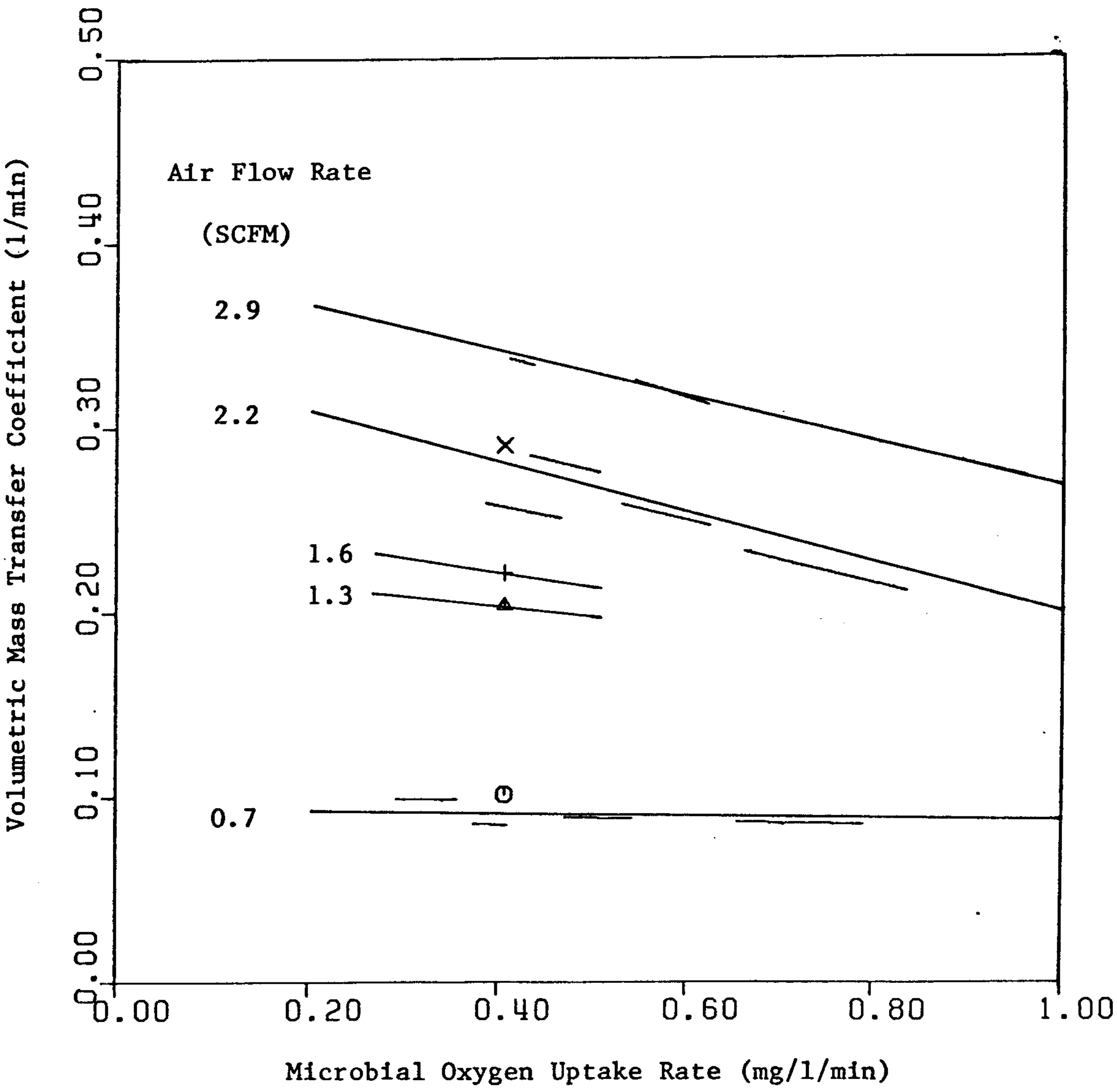


Figure 25. K_La vs. OUR at various Air Flow Rate in 10' Mixed Liquor

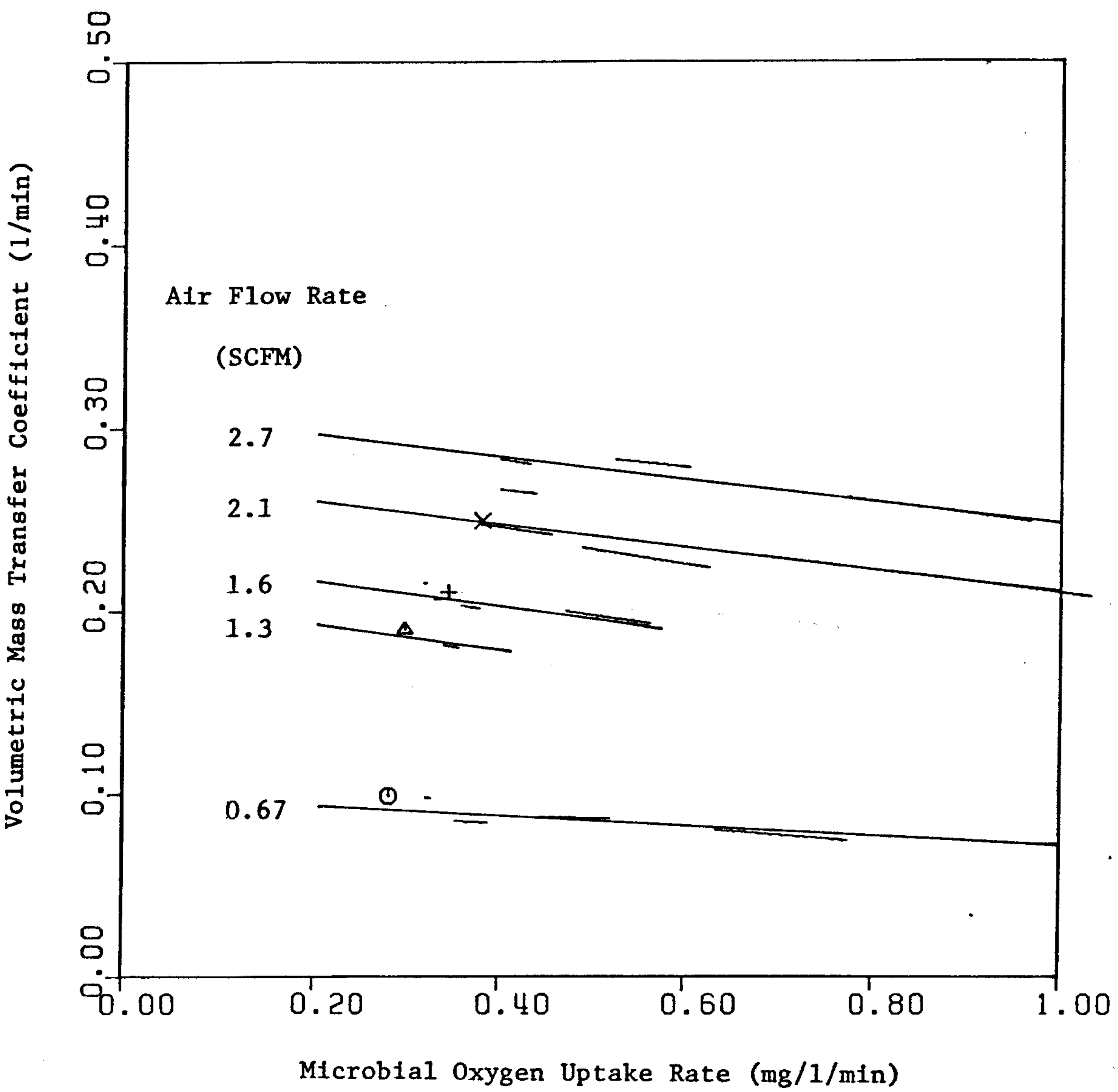


Figure 26. K_La vs. OUR at various Air Flow Rate in 5' Mixed Liquor

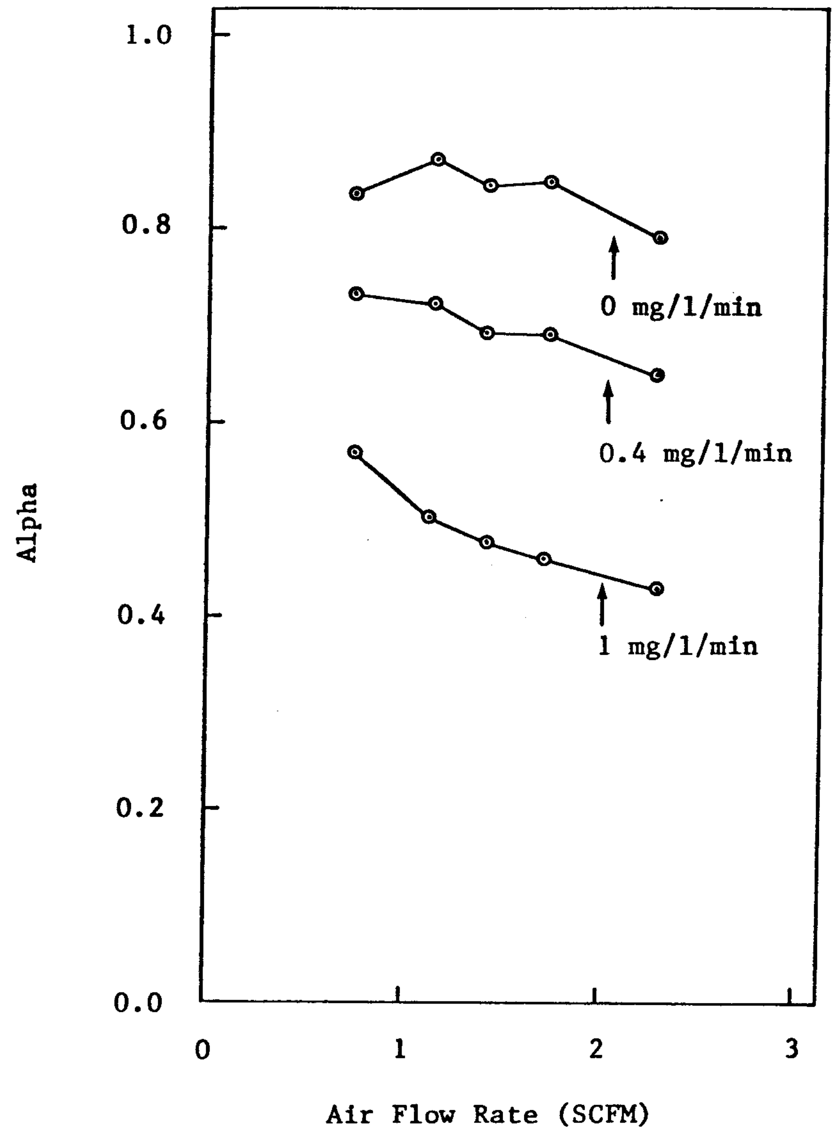


Figure 27. Alpha vs. Air Flow Rate in 14'2" Mixed Liquor Depth

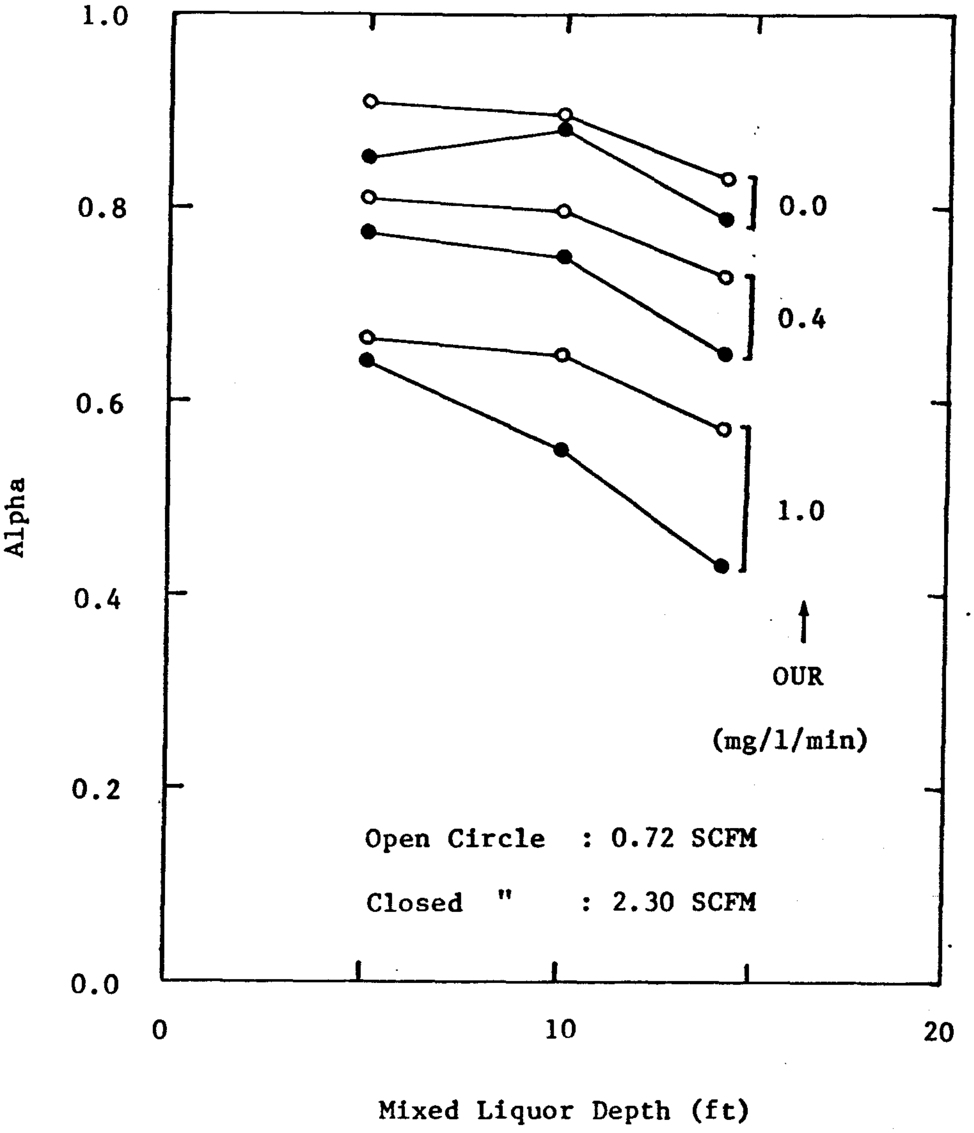


Figure 28. Alpha vs. Mixed Liquor Depth in 20" ID Aeration Column

A. Whittier Narrows Wastewater Reclamation Plant

(30' W x 100' L x 14.84' WD, 724 diffusers)

Air Flow Rate (SCFM/diffuser)	K_La (1/hr)	C^*_∞ (mg/l)
1.26	8.89	10.58
1.25	10.14	10.38
1.25	10.10	10.71
1.26	9.93	10.70
1.27	10.17	10.58
2.47	18.08	10.34
0.77	6.67	10.55

B. LACSD's Joint Water Pollution Control Plant

(20' W x 20' L x 25' D, 98 diffusers)

Air Flow	K_La	C^*_∞	Pressure Head (ft water)	
(scfm)	(1/hr)	(mg/l)	total	static
73.1	6.61	10.40	15.01	14.19
58.3	5.06	10.56	16.01	15.20
123.9	9.92	10.23	15.00	14.20
115.2	9.00	10.52	16.10	15.20
249.6	18.71	10.38	15.00	14.06
232.0	16.21	10.52	16.00	15.00

Table 7. Clean Water Test in Full Scale Systems

(After Yunt, 1983)

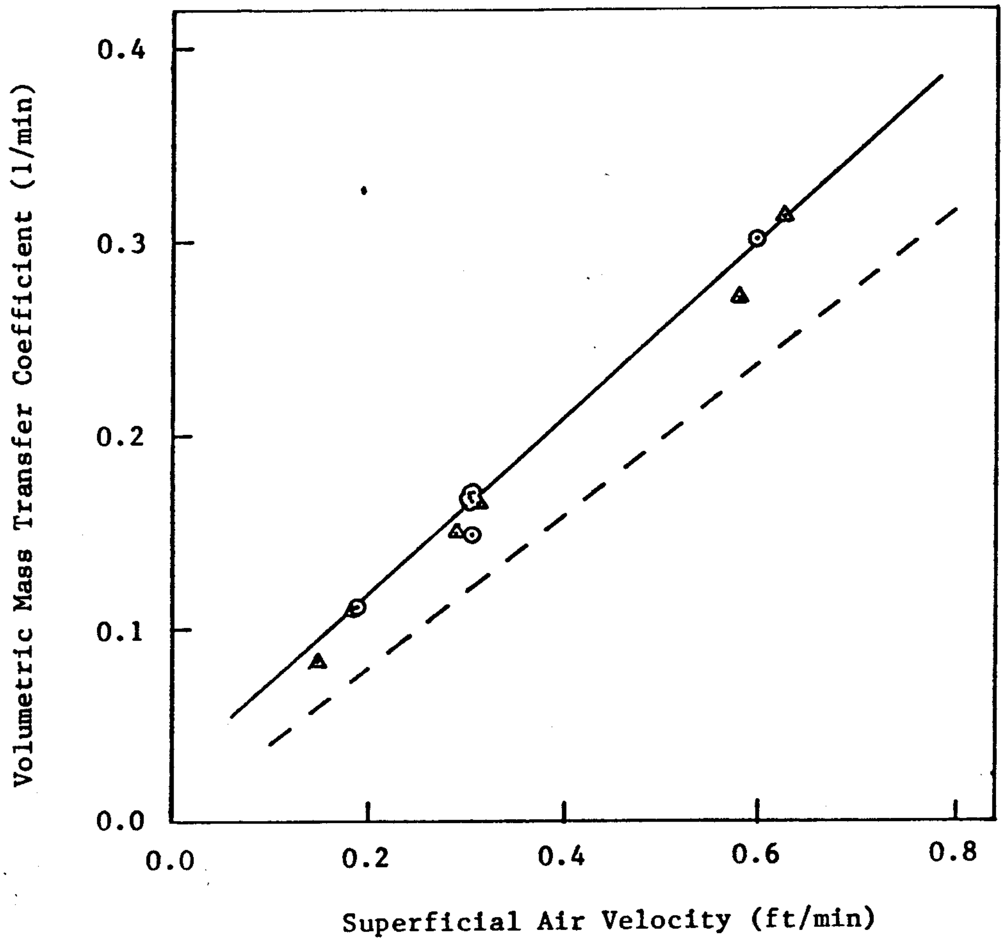


Figure 29. K_La vs. Superficial Air Flow Rate in Clean Water

results is presented in Table 8.

The mixed-liquor test result in column can be extrapolated to field condition as follows:

(1). Read off K_La values for given uptake rate and air flow rate from Figure 27.

(2). Convert the K_La value to that in field condition according to floor area per unit diffuser.

(3). Apply geometric conversion factor (defined as the ratio of K_La value in clean water in aeration column to the one in full scale tank).

Table 9 shows the detail of calculation and the measured value of column test result is given in Figure 30.

	Flow Rate		OUR	K_La	
run	Water	Air	(measured)	Dual NSS	SS (β=0.95)
	(MGD)	(SCFM)	(mg/l/hr)	(1/hr)	(1/hr)
1	6.0	520	35.4	3.7	4.7
		960		6.1	8.6
2	5.4	500	29.8	---	---
		1190		6.8	6.9
3	5.4	590	34.2	---	---
		1160		6.8	6.6
4	5.7	690	30.8	5.4	4.5
		1230		7.2	6.7
5	2.9	390	22.0	3.0	3.0
		1440		8.1	7.6
6	2.9	330	18.5	3.0	2.5
		1160		7.2	6.6
7	5.6	600	31.0	4.0	4.0
		1130		7.7	7.0

Table 8. Mixed Liquor Test in Full Scale System

(After Mueller et al., 1982)

Table 9. Extrapolation of Column Test Results to Full Scale

Run # (1)	G_s scfm diffuser (2)	OUR mg/l/min (3)	K_La 1/min (4)	K_La 1/min (5)	Alpha (6)
1	1.477	0.590	0.265	0.160	0.603
	2.727		0.490	0.277	0.565
2	1.420	0.497	-----	-----	-----
	3.381		0.608	0.355	0.584
3	1.676	0.570	-----	-----	-----
	3.295		0.593	0.330	0.557
4	1.960	0.513	0.352	0.210	0.596
	3.494		0.629	0.360	0.573
5	0.657	0.367	0.117	0.093	0.793
	2.424		0.436	0.282	0.647
6	0.938	0.308	0.168	0.128	0.762
	3.295		0.593	0.385	0.649
7	1.705	0.517	0.306	0.188	0.614
	3.210		0.577	0.335	0.580

(2) air flow rate per diffuser in mixed-liquor test.

(3) measured uptake rate in mixed-liquor.

(4) K_La in clean column test for given air flow rate per diffuser.

(5) K_La in mixed-liquor column test for given air flow rate per diffuser and oxygen uptake rate.

(6) column 6 = column 5 / column 4

(table 9 continue)

Area per Diffuser sq ft (7)	1/min (8)	Estimated K_La 1/min (9)	1/hr (10)	Measured K_La 1/hr (11)	1/hr (12)
8.523	0.105	0.063	3.79	3.7	4.7
	0.171	0.096	5.78	6.1	8.6
8.523	-----	-----	----	---	---
	0.205	0.120	7.18	6.8	6.9
8.523	-----	-----	----	---	---
	0.200	0.112	6.70	6.8	6.6
8.523	0.130	0.078	4.66	5.4	4.5
	0.211	0.121	7.25	7.2	6.7
5.051	0.085	0.068	4.06	3.0	3.0
	0.242	0.157	9.41	8.1	7.6
8.523	0.076	0.058	3.49	3.0	2.5
	0.200	0.130	7.81	7.2	6.6
8.523	0.117	0.072	4.30	4.0	4.0
	0.196	0.114	6.82	7.7	7.0

(7) surface area per unit diffuser.

(8) estimated from clean water full scale test.

(9) column 9 = column 6 x column 8

(10) column 10 = column 9 x 60

(11) DNSS test result from Table 8

(12) SS test result from Table 8

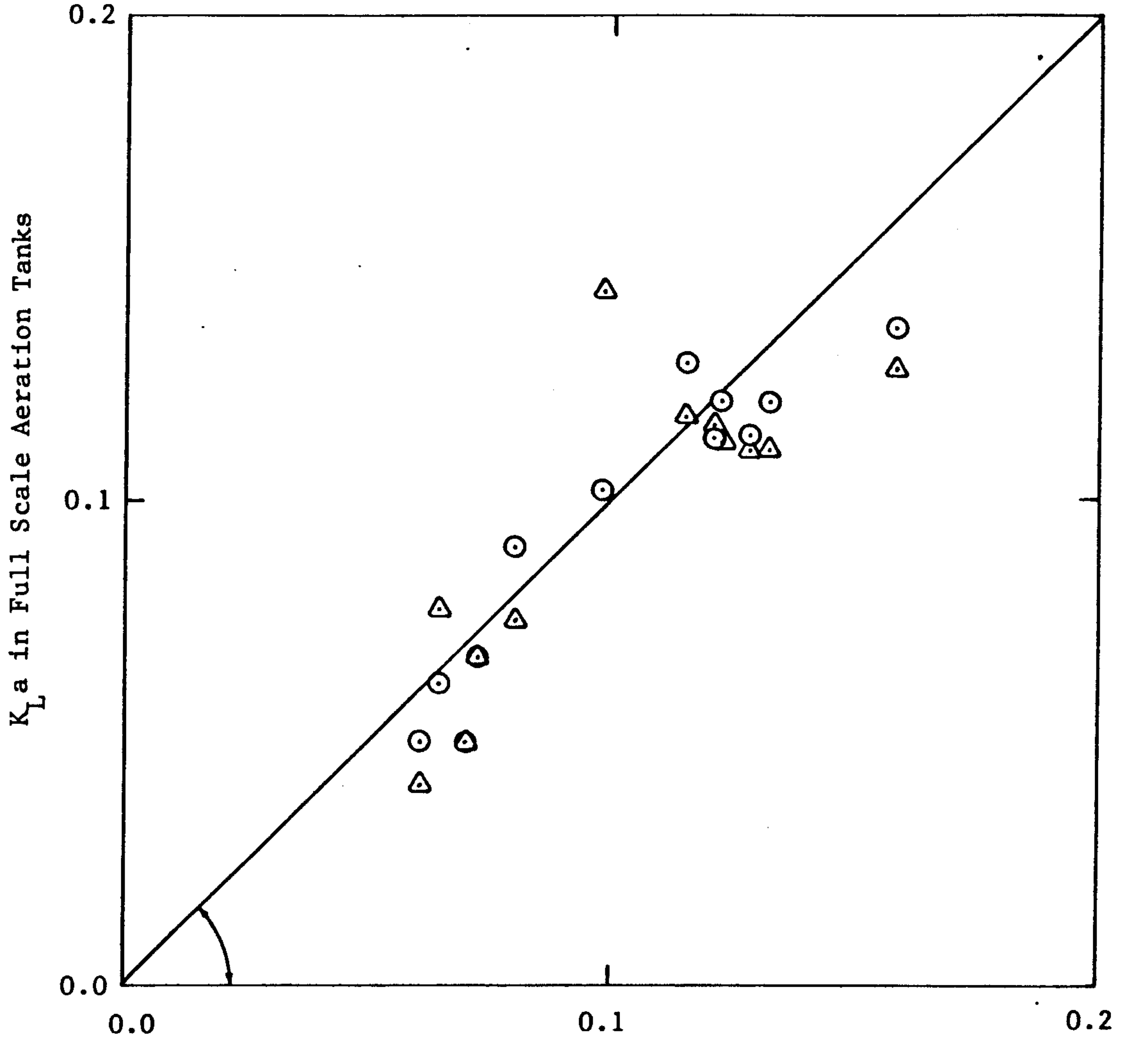
B. EXPERIMENTAL RESULTS OF SURFACE AERATION

For dimensional analysis of surface aeration total 61 experiments were performed with tap water and detergent water. The number of experiments for different tanks and detergent concentration are summarized in the Table 10.

Clean water test results in three tanks are shown in Figure 31. The dotted lines are least squares fits to data for each tank. Slope varies from 1.07 to 0.87, decreases with tank size. The intersections of three fitted line have same ratios as water volume, which indicates volumetric mass transfer coefficient can be related to power input per unit volume. The graph was plotted for K_La value versus power input per one million gallon as in Figure 32. The data in Figure 32 have a common slope of 0.97 with correlatiion coefficient of 0.9914. To determine the dependency of K_La on surface tension, the following data from the 190 gallon water volume test have been used. The value of 0.88 for exponent of surface tension for K_La.

From these relations, the following expression has been obtained as a nondimensional expression. Figure 33 shows a good agreement between the data and the equation 85.

$$\frac{Y_H}{Ne^{0.97}} \cdot We^{0.8} = 2.36 \times 10^{-7} \times Fr^{2.22} \tag{85}$$



Extrapolated Value of $K_L a$ from Column Test Results

Figure 30. Comparison of 20-in column and Full Scale Tank Test Results

DSS Conc. \ tank	30	55	200
0.	8	6	7
1	--	4	--
3	4	3	5
5	3	4	4
10	5	--	4
15	--	--	4

Table 10. Number of Experiments of Surface Aeration

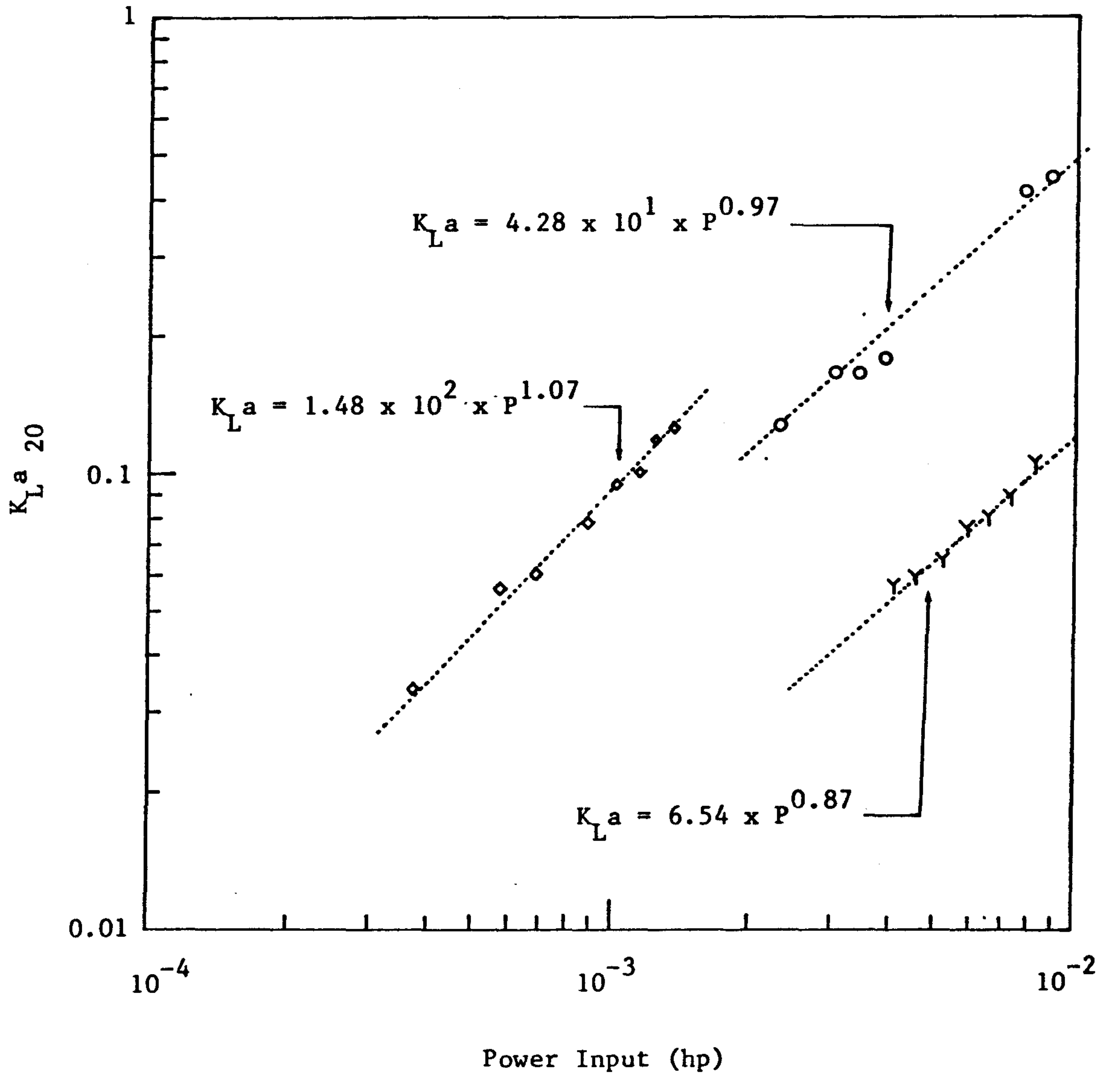


Figure 31. K_La vs. Power Input in Surface Aeration Tanks

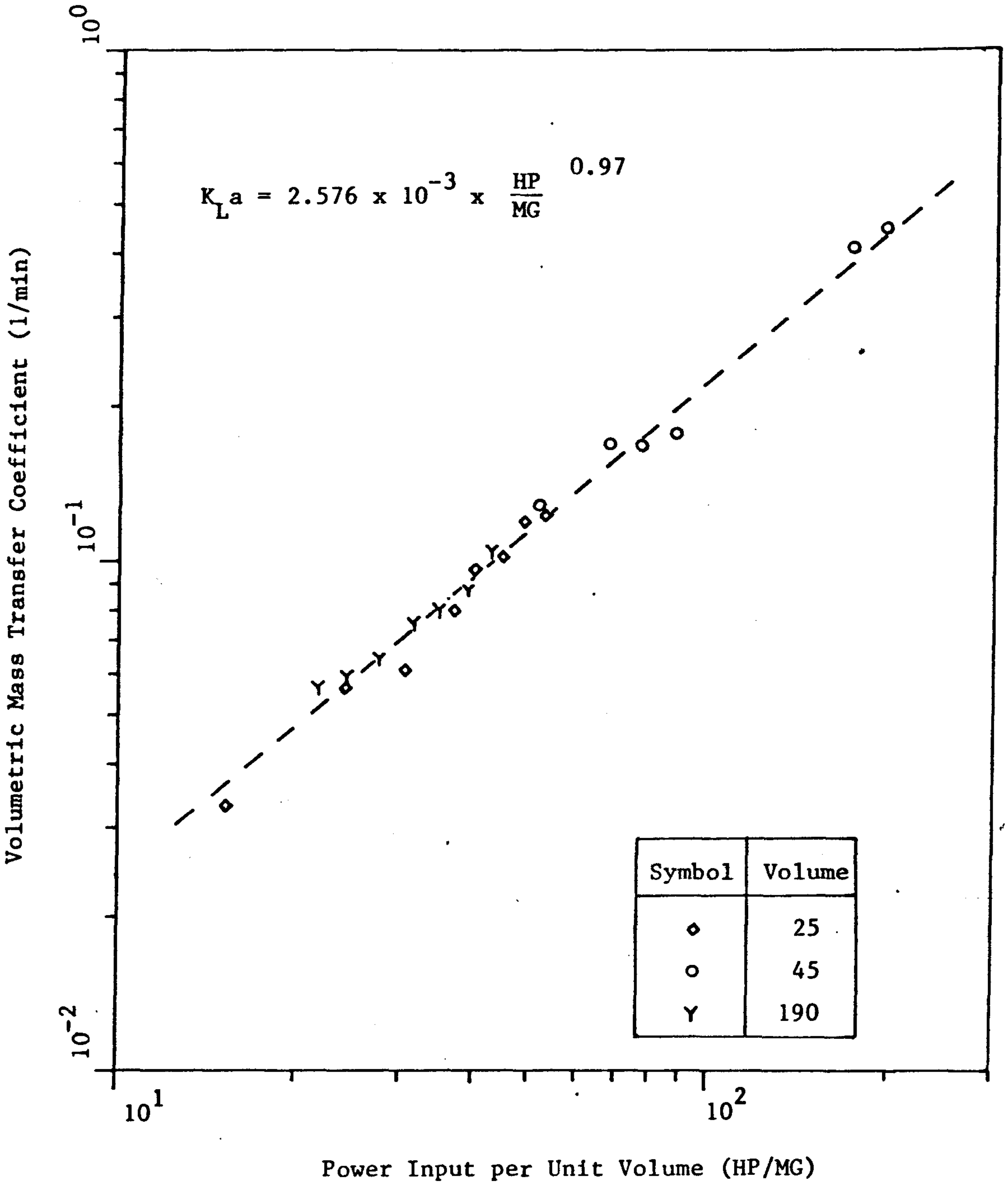


Figure 32. K_La vs. HP/MG in Surface Aeration

Surface Tension (dynes/cm)	Power Input (x 1000 hp)	K_La (1/min)
70.4	5.80	0.0710
65.5	5.87	0.0674
62.5	5.72	0.0621
56.2	5.80	0.0600

Table 11. K_La vs. Surface Tension at Constant Power Input

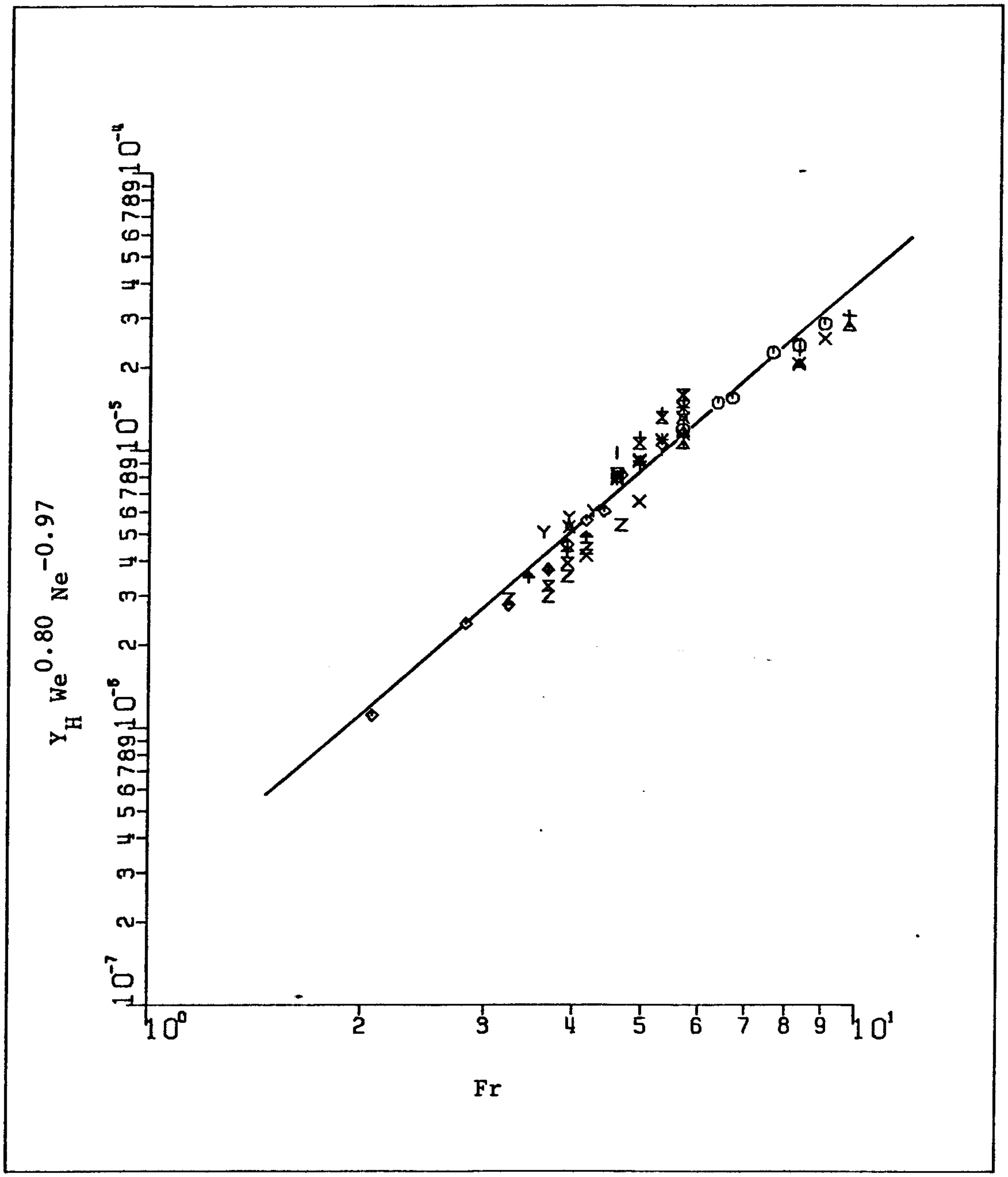


Figure 33. Nondimensional presentation of Surface Aeration Data

VII. CONCLUSIONS AND RECOMMENDATIONS

In this research, the oxygen transfer mechanism in the air-water interface has been studied using two aeration methods, diffused and surface aeration. Based on mathematical development and experimental work the primary objective has been achieved.

For diffused aeration, mathematical models which can handle rapidly changing water quality under process condition have been derived and their methods for parameter estimation have been developed. The models are verified their applicability experimentally. It is shown that the pilot scale aeration column with full scale liquid depth under operating condition can be used to simulate the full scale aeration tank. The experiments were conducted in tap water, surfactant water and mixed-liquor. The variation of alpha factor in the mixed-liquor can be explained with readily measurable variable, microbial oxygen uptake.

From the experimental results, the following conclusions are made:

(1). Volumetric mass transfer coefficient is linearly dependent on air flow rate.

(2). Volumetric mass transfer coefficient generally decreases with increasing liquid depth. However, when the clearance between floor and diffuser is large (2 ft in this research), the trend can be reversed, i.e., at shallower liquid depth, K_La is lower.

(3). Alpha factor in detergent water for fine bubble diffusers varies between 0.4 to 0.8, with same concentration of surfactant. It is believed that the presence of mixed-liquor residue changes the efficiency of surfactant at the air-water interface.

(4). Microbial oxygen uptake rate in mixed-liquor can be used to explained the variation of volumetric oxygen mass trasnfer coefficient in mixed-liquor. As the water is treated, soluble substrate is removed and microbial oxygen uptake rate decreases to the value of endogeneous phase and the volumetric oxygen mass transfer coefficient increases. It is shown the variation of volumetric oxygen mass transfer coefficient can be related linearly to microbial oxygen uptake rate.

(5). Alpha factors in the mixed-liquor increase as the water is treated.

(6). Alpha factors in the surfactant water and mixed-liquor decrease with increasing air flow rate.

(7). Alpha factors in the mixed-liquor increase with decreasing liquid depth.

For surface aeration method, similarity theory has been applied to derived nondimensional parameters.

Three sizes of laboratory scale aeration tank have been used. Water characteristics are changed using 5 mg/l of anionic surfactant, dodecyl sodium sulfate. The following statements can be drawn from surface aeration experiments.

(1). Volumetric mass transfer coefficient is linearly dependent on the power input per unit volume.

(2). The transfer number, which descrived the mass transfer rate of oxygen, varies with 0.88 power of Weber number.

(3). The transfer number varies with 2.22 power of Froud number.

RECOMMENDATIONS

The following recommendation can be made from this research.

(1). For better understanding of the role of surfactant in the oxygen transfer mechanism, it is required to investigate the effect of dynamic surface tension on the air-water interface.

(2). For the mixed-liquor test in aeration column, more extensive experiments are required. Mixed-liquor suspended solid (MLSS) or sludge retention time might change the relationship of volumetric mass transfer coefficients and microbial oxygen uptake rate.

(3). Great care must be used if $K_L a$ versus R(t) data must be extrapolated beyond the experimental range observed here.

APPENDIX

A. OXYGEN UPTAKE RATE DATA

9/28/81		9/29/81		9/30/81	
Time	OUR	Time	OUR	Time	OUR
0.0	37.38	5.4	22.75	65.0	24.86
12.0	29.29	19.0	21.60	82.5	25.65
22.0	28.90	24.5	22.4	96.0	23.85
31.0	28.06	42.5	21.60	105.5	24.53
41.0	27.82	52.0	22.10	115.0	23.63
52.0	27.45	61.0	20.74	128.0	23.40
61.0	26.91	71.0	20.18	141.5	24.19
77.0	26.30	82.5	19.62	152.0	24.64
91.0	25.82	105.0	19.17	175.0	24.30
103.0	26.04	115.0	19.84	175.0	24.30
115.5	25.93	129.0	19.73	188.0	23.06
128.0	25.59	154.0	19.95	203.0	20.93
138.0	25.24	166.0	19.73	220.7	19.57
		178,0	19.62	240.0	19.47
		188.5	18.71	254.0	19.24
		200.0	19.05	266.0	18.79
		212.5	18.71	278.0	18.11
		224.0	18.75	288.5	18.00
				300.0	17.21
				310.0	17.78
				321.0	17.55
				332.0	16.82
				344.0	16.71
				354.0	15.98

(*) indicates the value at the pumping location in Tank 1

12/ 7/81		12/ 9/81		12/10/81	
Time	OUR	Time	OUR	Time	OUR
12.0	31.40	53.5	56.62*	7.2	38.09*
23.4	27.43	60.7	55.11*	13.0	41.60*
34.5	25.24	66.8	53.96*	21.3	38.99*
47.5	23.78	83.5	40.18	29.8	25.06
67.4	23.67	89.6	40.65	46.9	23.76
81.0	22.42	97.6	36.67	60.2	21.73
97.9	21.44	106.8	34.36	71.7	20.91
103.5	20.57	115.8	30.73	84.1	20.30
133.5	20.21	126.9	26.40	100.9	19.27
149.4	18.76	139.0	25.02	118.1	18.85
169.9	17.46	155.5	21.27	134.8	18.29
		169.1	19.77	147.3	17.81
		186.0	18.24	158.4	17.21
				171.3	16.65
				180.1	70.59
				184.6	66.77
				189.5	63.18
				203.2	27.87
				214.3	23.79
				226.3	21.84
				244.4	29.05

(*) indicates the value at the pumping location in Tank 1

12/28/81		12/29/81		3/31/82	
Time	OUR	Time	OUR	Time	OUR
24.0	34.10*	6.0	43.42*	4.4	85.75*
37.0	32.83*	12.5	41.24*	11.3	67.99*
57.0	32.48*	20.0	42.85*	14.7	79.78*
71.5	32.83*	27.5	36.87	22.4	81.73
83.5	24.48	38.5	31.70	34.6	62.26
95.0	23.11	47.5	29.29	49.6	53.95
106.5	21.50	57.5	26.99	59.3	48.56
116.0	21.96	69.5	24.93	68.8	44.32
126.5	20.70	78.0	23.78	79.5	40.80
140.5	19.33	88.5	22.86	94.9	36.17
153.5	18.64	98.5	21.71	113.4	33.01
165.0	19.10	108.5	21.71	119.1	34.31
175.0	17.96	117.0	22.17	151.5	31.10
185.0	18.64	126.5	20.68	156.1	31.25
197.0	18.18	140.5	20.56	160.8	29.93
206.5	18.42	156.5	19.87		
216.5	17.50	168.0	19.41		
226.5	17.50	176.5	19.41		
236.0	17.16	183.5	18.95		
247.0	17.16	192.5	19.07		
264.0	16.59	204.5	18.93		

(*) indicates the value at the pumping location in Tank 1

4/ 5/82		4/ 6/82 (1)		4/ 6/82 (2)	
Time	OUR	Time	OUR	Time	OUR
7.1	43.00*	9.2	48.84*	8.8	62.29*
15.5	42.74*	17.2	48.79*	12.8	60.43*
23.6	42.27*	22.7	47.39*	17.7	62.50*
30.3	43.93	36.1	50.46	26.5	64.84
39.6	42.11	45.3	43.58	32.6	54.84
50.4	37.48	55.8	39.70	41.1	53.52
61.8	32.82	66.3	36.86	50.0	46.42
77.4	27.28	72.7	32.85	60.8	41.84
89.9	24.50	77.9	36.81	72.9	36.05
100.0	22.60	84.7	29.75	81.6	33.70
107.1	20.46	90.7	30.86	93.7	31.46
116.6	19.81	100.0	25.85	102.8	27.98
130.4	18.53	105.2	26.73	111.6	27.32
				123.4	26.14
				135.7	25.69
				146.2	22.76
				156.3	22.95

(*) indicates the value at the pumping location in Tank 1

4/ 7/82 (1)		4/ 7/82 (2)		4/20/82	
Time	OUR	Time	OUR	Time	OUR
11.0	62.79*	8.2	67.12*	13.4	51.31*
15.6	59.49*	13.8	64.48*	19.9	52.96*
21.0	60.47*	18.7	68.66*	27.3	50.48
34.1	62.77	28.4	60.66	35.5	45.45
44.5	56.49	33.9	56.45	45.3	40.69
50.2	48.45	40.0	51.24	52.8	36.46
56.4	45.12	42.8	51.22	59.7	34.59
62.9	42.19	48.3	46.68	67.8	31.75
69.2	34.78	51.3	47.29	75.0	30.23
76.7	32.80	56.8	41.10	88.3	28.45
83.7	29.55	68.7	36.16	109.9	26.00
92.1	27.19	71.2	36.27	115.8	24.27
101.6	26.77	78.7	32.87	130.1	22.78
		87.0	30.30		
		95.0	27.58		
		98.3	28.01		
		108.2	27.08		
		113.5	26.81		
		124.2	24.04		

(*) indicates the value at the pumping location in Tank 1

4/21/82		4/22/82	
Time	OUR	Time	OUR
4.1	69.49*	8.0	72.35*
10.0	68.18*	13.0	65.88*
17.5	68.06*	19.0	64.34*
23.1	67.95	26.0	65.58
31.3	57.28	30.2	64.60
38.7	54.02	38.7	55.45
48.0	48.16	44.2	52.16
52.6	44.09	57.8	45.93
55.9	43.77	65.8	42.48
60.9	42.66	77.8	38.28
63.1	40.36	84.1	41.56
65.6	38.66	92.0	41.90
70.6	36.23	99.8	40.55
73.3	34.49	110.2	39.15
78.0	32.85	123.4	40.99
		128.8	40.78
		137.6	40.56
		151.1	37.36
		161.8	38.89
		172.0	38.74

(*) indicates the value at the pumping location in Tank 1

B. MIXED LIQUOR TEST DATA

Constant Oxygen Uptake Rate

Run	Date	Liquid Depth	Liquid Temp.	Air Flow Rate	R	K_La_{20}	$C^*_{\infty 20}$
48	9/28	14.2	28.9	2.24	28.2	0.254	9.21
49		14.2	28.9	1.68	27.0	0.202	9.25
50		14.2	28.9	1.41	25.5	0.179	9.37
51		14.2	28.2	0.84	25.3	0.091	9.40
52	9/29	14.2	26.8	2.32	21.0	0.280	9.73
53		14.2	26.7	1.73	20.0	0.207	9.80
54		14.2	26.7	1.44	19.6	0.181	9.84
55		14.2	26.6	1.14	19.8	0.154	9.80
56		14.2	26.6	0.72	18.7	0.096	9.69
57	9/30	10.0	27.2	2.21	24.5	0.290	9.42
58		10.0	27.2	1.60	24.5	0.221	9.58
59		10.0	27.3	1.33	24.5	0.204	9.46
60		10.0	27.5	0.67	24.5	0.101	10.05
62		4.9	27.3	1.27	18.0	0.190	9.32
63		4.9	27.2	0.64	17.0	0.099	9.08
64	12/ 7	14.2	23.6	2.32	25.0	0.265	10.40
67		14.1	23.4	1.73	19.0	0.239	9.66
71	12/10	5.0	23.4	2.10	23.0	0.249	8.85
72		5.0	23.3	1.57	20.8	0.211	8.80

Exponential Decaying Oxygen Uptake Rate

Run	Date	Depth	Temp	Air Flow Rate	OUR Parameters			Testing Time	
					Ku	Ro	Rc	Start	End
65	12/ 7	14.15	23.6	1.43	0.0155	15.3	17.5	62.7	86.7
66		14.15	23.5	0.71	0.0155	15.3	17.5	94.2	124.2
70	12/ 9	10.03	23.9	0.70	0.0127	95.9	8.7	159.0	187.0
75	12/10	10.00	23.3	1.67	0.0361	12.5	16.7	2.5	17.5
76	12/28	9.90	22.1	1.65	0.0155	28.4	16.6	86.5	101.5
77		9.90	22.1	2.23	0.0155	28.4	16.6	114.0	126.0
78		9.85	22.0	1.67	0.0155	28.4	16.6	137.4	152.4
79		9.80	21.9	1.39	0.0155	28.4	16.6	162.0	177.0
80		9.80	21.8	1.67	0.0155	28.4	16.6	189.0	204.0
81		9.75	21.7	0.70	0.0155	28.4	16.6	214.0	238.0
82		9.75	21.6	1.67	0.0155	28.4	16.6	244.0	259.0
83	12/29	5.06	22.1	1.56	0.0252	34.5	19.1	34.0	52.0
84		5.06	22.1	2.09	0.0252	34.5	19.1	61.5	76.5
85		5.06	22.0	1.57	0.0252	34.5	19.1	88.5	103.5
86		5.06	21.9	1.31	0.0252	34.5	19.1	106.5	126.5
87		5.06	21.8	1.57	0.0252	34.5	19.1	133.5	148.5
88		5.06	21.6	0.66	0.0252	34.5	19.1	161.0	187.0
89		5.06	21.5	1.58	0.0252	34.5	19.1	193.5	208.5
90	3/31	14.16	22.6	2.30	0.0200	62.5	27.8	29.0	64.0
92		14.13	22.7	2.33	0.0200	62.5	27.8	100.0	127.0
94		14.10	22.6	2.33	0.0200	62.5	27.8	156.0	173.0
91		14.15	22.7	0.73	0.0200	62.5	27.8	72.0	99.0
93		14.12	22.6	0.73	0.0200	62.5	27.8	128.0	150.0
95		14.10	22.6	0.73	0.0200	62.5	27.8	174.0	186.0
96	4/ 5	14.30	22.1	2.32	0.0129	55.2	7.9	40.0	64.0
97		14.29	22.2	2.33	0.0129	55.2	7.9	75.0	97.0
98		14.25	22.3	2.32	0.0129	55.2	7.9	106.0	126.0
99	4/6-1	10.14	22.4	2.23	0.0143	61.2	12.9	34.5	57.5
100		10.14	22.5	2.22	0.0143	61.2	12.9	63.5	81.5
101		10.12	22.6	2.21	0.0143	61.2	12.9	87.0	107.0
102	4/6-2	10.06	22.6	2.90	0.0207	74.8	20.1	33.0	48.0
104		10.06	22.6	2.89	0.0207	74.8	20.1	70.5	85.5
106		10.00	22.5	2.91	0.0207	74.8	20.1	119.5	133.5
103		10.06	22.6	0.69	0.0207	74.8	20.1	48.5	65.5
105		10.06	22.6	0.69	0.0207	74.8	20.1	86.0	106.0
107		10.00	22.4	0.70	0.0207	74.8	20.1	134.5	165.5

108	4/7-1	5.03	22.8	2.12	0.0217	106.6	13.3	36.0	62.0
109		5.03	22.9	2.06	0.0217	106.6	13.3	68.0	87.0
110		5.03	22.9	2.12	0.0217	106.6	13.3	93.0	111.0
111	4/7-2	5.00	22.6	2.70	0.0203	76.5	17.8	31.5	48.5
113		5.00	22.6	2.72	0.0203	76.5	17.8	70.0	85.0
115		5.00	22.5	2.73	0.0203	76.5	17.8	110.0	123.0
112		5.00	22.6	0.67	0.0203	76.5	17.8	48.5	65.5
114		5.00	22.6	0.69	0.0203	76.5	17.8	86.0	106.0
116		5.00	22.4	0.65	0.0203	76.5	17.8	129.5	152.5
117	4/20	14.23	24.2	2.24	0.0238	56.7	20.9	43.5	64.5
118		14.23	24.6	2.24	0.0238	56.7	20.9	72.0	92.0
119		14.23	24.8	2.24	0.0238	56.7	20.9	100.0	118.0

(continue)

Run	OUR During Test				K_La_{20}		$C^*_{\infty 20}$
	Start	End	a_c	b_c	Start	End	
65	0.387	0.358	-0.108	0.234	0.192	0.196	9.73
66	0.350	0.328	-0.011	0.104	0.100	0.100	9.61
70	0.357	0.293	-0.003	0.100	0.099	0.099	9.34
75	0.468	0.388	-0.095	0.296	0.251	0.259	8.86
76	0.400	0.374	-0.146	0.310	0.252	0.255	9.49
77	0.357	0.343	-0.114	0.383	0.343	0.344	9.33
78	0.332	0.321	-0.142	0.336	0.289	0.291	9.15
79	0.314	0.306	-0.081	0.261	0.235	0.236	9.18
80	0.301	0.296	-0.134	0.317	0.277	0.278	9.29
81	0.293	0.288	-0.006	0.133	0.131	0.131	9.28
82	0.287	0.285	-0.123	0.329	0.294	0.294	9.15
83	0.563	0.473	-0.073	0.234	0.193	0.200	8.73
84	0.440	0.402	-0.051	0.287	0.264	0.266	8.93
85	0.380	0.361	-0.069	0.228	0.201	0.203	8.97
86	0.358	0.342	-0.079	0.208	0.180	0.181	8.85
87	0.338	0.332	-0.053	0.224	0.206	0.207	9.03
88	0.328	0.324	-0.013	0.102	0.098	0.098	8.98
89	0.323	0.321	-0.081	0.241	0.215	0.216	8.96

90	1.047	0.753	-0.156	0.332	0.168	0.214	9.34
92	0.605	0.546	-0.199	0.353	0.233	0.244	9.38
94	0.510	0.497	-0.169	0.335	0.249	0.251	9.39
91	0.711	0.608	-0.023	0.101	0.085	0.088	9.47
93	0.544	0.516	-0.023	0.102	0.090	0.090	9.60
95	0.496	0.489	-0.029	0.105	0.090	0.091	9.59
96	0.680	0.534	-0.154	0.328	0.223	0.246	9.51
97	0.480	0.394	-0.173	0.305	0.222	0.236	9.72
98	0.365	0.312	-0.147	0.317	0.263	0.271	9.51
99	0.837	0.663	-0.121	0.313	0.212	0.233	9.46
100	0.626	0.533	-0.127	0.326	0.247	0.259	9.36
101	0.509	0.435	-0.120	0.337	0.276	0.285	9.07
102	0.964	0.796	-0.120	0.389	0.274	0.294	9.28
104	0.624	0.547	-0.169	0.418	0.313	0.326	9.34
106	0.439	0.413	-0.140	0.395	0.334	0.338	9.39
103	0.791	0.656	-0.006	0.090	0.086	0.086	9.40
105	0.544	0.473	-0.006	0.092	0.089	0.089	9.39
107	0.411	0.375	-0.014	0.091	0.085	0.086	9.52
108	1.034	0.684	-0.064	0.275	0.208	0.231	8.73
109	0.627	0.490	-0.083	0.276	0.224	0.235	8.86
110	0.457	0.381	-0.077	0.277	0.242	0.248	8.82
111	0.970	0.774	-0.067	0.315	0.249	0.262	8.83
113	0.605	0.525	-0.045	0.306	0.279	0.283	8.81
115	0.434	0.402	-0.064	0.309	0.281	0.283	8.87
112	0.774	0.635	-0.037	0.104	0.075	0.080	8.62
114	0.520	0.446	-0.009	0.092	0.087	0.088	8.48
116	0.389	0.355	-0.024	0.094	0.085	0.085	8.82
117	0.685	0.553	-0.146	0.325	0.225	0.245	9.57
118	0.519	0.455	-0.150	0.329	0.251	0.260	9.52
119	0.437	0.406	-0.142	0.316	0.254	0.258	9.61

C. SURFACE AERATION DATA

Run	SAA (mg/l)	Vol (gal)	Water (°c)	Dia (iN)	Sub (in)	RPM
1- 1	0.0	45.	27.5	2.0	3.1	2400
1- 2	0.0	45.	27.5	2.0	3.1	2100
1- 3	0.0	45.	27.5	2.0	3.1	2300
1- 4	0.0	45.	26.0	3.5	3.8	1500
1- 5	0.0	45.	26.5	3.5	4.0	1625
1- 6	0.0	45.	25.8	2.0	3.1	2500
1- 7	1.0	45.	28.5	2.0	3.1	2600
1- 8	1.0	45.	27.0	2.0	3.1	2400
1- 9	1.0	45.	28.0	3.5	3.9	1500
1-10	1.0	45.	29.0	3.5	3.9	1500
1-11	3.0	45.	26.0	2.0	3.1	2600
1-12	3.0	45.	25.6	2.0	3.1	2400
1-13	3.0	45.	25.2	3.5	3.9	1500
1-14	5.0	45.	27.0	2.0	3.1	2400
1-15	5.0	45.	26.0	2.0	3.1	2500
1-16	5.0	45.	26.0	3.5	3.7	1500
1-17	5.0	45.	28.8	3.5	3.9	1400
2- 1	0.0	25.	24.0	2.0	3.0	1200
2- 2	0.0	25.	24.5	2.0	3.0	1400
2- 3	0.0	25.	25.0	2.0	3.0	1500
2- 4	0.0	25.	27.0	2.0	3.0	1600
2- 5	0.0	25.	28.0	2.0	3.0	1650
2- 6	0.0	25.	25.0	2.0	3.0	1700
2- 7	0.0	25.	28.5	2.0	3.0	1750
2- 8	0.0	25.	22.5	2.0	3.0	1800
2- 9	3.0	25.	25.5	2.0	3.0	1700
2-10	3.0	25.	25.4	2.0	3.0	1650
2-11	3.0	25.	25.8	2.0	3.0	1600
2-12	3.0	25.	25.6	2.0	3.0	1550
2-13	5.0	25.	27.7	2.0	3.0	1700
2-14	5.0	25.	25.0	2.0	3.0	1650
2-15	5.0	25.	24.5	2.0	3.0	1600
2-16	10.0	25.	26.3	2.0	3.0	1500
2-17	10.0	25.	28.3	2.0	3.0	1600
2-18	10.0	25.	28.5	2.0	3.0	1650
2-19	10.0	25.	27.0	2.0	3.0	1700
2-20	10.0	25.	26.6	2.0	3.0	1800

3- 1	0.0	190.	24.0	3.5	5.5	1200
3- 2	0.0	190.	24.0	3.5	5.5	1250
3- 3	0.0	190.	26.3	3.5	5.5	1300
3- 4	0.0	190.	24.6	3.5	5.5	1350
3- 5	0.0	190.	25.0	3.5	5.5	1400
3- 6	0.0	190.	24.9	3.5	5.5	1450
3- 7	0.0	190.	24.4	3.5	5.5	1500
3- 8	3.0	190.	23.8	3.5	5.5	1250
3- 9	3.0	190.	23.5	3.5	5.5	1350
3-10	3.0	190.	22.8	3.5	5.5	1400
3-11	3.0	190.	22.5	3.5	5.5	1450
3-12	3.0	190.	22.9	3.5	5.5	1500
3-13	5.0	190.	22.4	3.5	5.5	1350
3-14	5.0	190.	22.6	3.5	5.5	1400
3-15	5.0	190.	22.5	3.5	5.5	1450
3-16	5.0	190.	22.3	3.5	5.5	1500
3-17	10.0	190.	22.1	3.5	5.5	1350
3-18	10.0	190.	21.2	3.5	5.5	1400
3-19	10.0	190.	21.1	3.5	5.5	1450
3-20	10.0	190.	20.9	3.5	5.5	1500
3-21	15.0	190.	21.4	3.5	5.5	1350
3-23	15.0	190.	21.0	3.5	5.5	1400
3-23	15.0	190.	20.8	3.5	5.5	1450
3-24	15.0	190.	20.6	3.5	5.5	1500

Run	K_LA_T (1/min)	K_LA_{20} (1/min)	Power (HP)	$C^*_{\infty 20}$ (mg/l)
1- 1	0.2001	0.1675	0.3455E-02	8.64
1- 2	0.1538	0.1287	0.2312E-02	8.25
1- 3	0.2016	0.1687	0.3043E-02	8.20
1- 4	0.4722	0.4096	0.7777E-02	8.01
1- 5	0.5209	0.4465	0.9237E-02	8.19
1- 6	0.2032	0.1771	0.3933E-02	8.03
1- 7	0.2146	0.1754	0.4379E-02	7.62
1- 8	0.1644	0.1392	0.3455E-02	7.69
1- 9	0.5065	0.4190	0.7648E-02	8.14
1-10	0.4668	0.3771	0.7402E-02	7.74
1-11	0.1932	0.1676	0.4379E-02	8.37
1-12	0.1614	0.1413	0.3442E-02	8.12
1-13	0.3865	0.3416	0.7527E-02	8.31
1-14	0.1518	0.1286	0.3455E-02	7.71
1-15	0.1665	0.1444	0.3958E-02	7.97
1-16	0.3930	0.3409	0.7402E-02	8.41
1-17	0.3070	0.2492	0.6205E-02	7.71
2- 1	0.0365	0.0332	0.3818E-03	8.93
2- 2	0.0626	0.0563	0.6045E-03	8.57
2- 3	0.0688	0.0611	0.7576E-03	8.96
2- 4	0.0941	0.0797	0.9212E-03	9.08
2- 5	0.1160	0.0960	0.1000E-02	8.92
2- 6	0.1146	0.1018	0.1116E-02	8.72
2- 7	0.1456	0.1190	0.1215E-02	8.71
2- 8	0.1303	0.1228	0.1323E-02	7.79
2- 9	0.0933	0.0819	0.1069E-02	8.65
2-10	0.0871	0.0766	0.9917E-03	8.81
2-11	0.0814	0.0710	0.9172E-03	8.63
2-12	0.0770	0.0674	0.8024E-03	8.83
2-13	0.0901	0.0752	0.1095E-02	8.89
2-14	0.0745	0.0663	0.9917E-03	9.08
2-15	0.0622	0.0559	0.8970E-03	9.01
2-16	0.0619	0.0533	0.7159E-03	8.46
2-17	0.0663	0.0545	0.8970E-03	8.75
2-18	0.0762	0.0623	0.9917E-03	8.72
2-19	0.0824	0.0698	0.1069E-02	8.98
2-20	0.0927	0.0792	0.1323E-02	8.48

3- 1	0.0622	0.0566	0.4121E-02	8.90
3- 2	0.0653	0.0594	0.4605E-02	8.88
3- 3	0.0749	0.0645	0.5223E-02	8.91
3- 4	0.0841	0.0755	0.5949E-02	8.60
3- 5	0.0900	0.0799	0.6597E-02	8.69
3- 6	0.0979	0.0871	0.7396E-02	8.72
3- 7	0.1158	0.1043	0.8110E-02	8.92
3- 8	0.0561	0.0513	0.4605E-02	8.97
3- 9	0.0732	0.0674	0.5874E-02	8.75
3-10	0.0776	0.0726	0.6558E-02	8.83
3-11	0.0846	0.0799	0.7276E-02	8.77
3-12	0.1004	0.0937	0.8110E-02	8.46
3-13	0.0658	0.0621	0.5724E-02	8.73
3-14	0.0720	0.0677	0.6480E-02	8.82
3-15	0.0822	0.0775	0.7276E-02	8.71
3-16	0.0987	0.0935	0.7943E-02	8.39
3-17	0.0592	0.0563	0.5574E-02	9.07
3-18	0.0698	0.0678	0.6403E-02	8.95
3-19	0.0811	0.0790	0.7122E-02	8.82
3-20	0.0904	0.0885	0.7777E-02	8.78
3-21	0.0627	0.0606	0.5574E-02	8.82
3-23	0.0671	0.0655	0.6325E-02	8.67
3-23	0.0759	0.0745	0.7034E-02	8.71
3-24	0.0823	0.0811	0.7693E-02	8.93

Surface Aeration Data (continue)

RUN	Froude #	Weber #	Power #	Transfer #
1- 1	0.8288E+01	0.2979E+04	0.1190E+00	0.5023E-08
1- 2	0.6346E+01	0.2281E+04	0.1189E+00	0.3861E-08
1- 3	0.7612E+01	0.2736E+04	0.1191E+00	0.5061E-08
1- 4	0.5666E+01	0.6238E+04	0.6684E-01	0.7913E-09
1- 5	0.6649E+01	0.7320E+04	0.6244E-01	0.8468E-09
1- 6	0.8993E+01	0.3233E+04	0.1198E+00	0.5658E-08
1- 7	0.9727E+01	0.3537E+04	0.1186E+00	0.5074E-08
1- 8	0.8288E+01	0.3014E+04	0.1190E+00	0.4254E-08
1- 9	0.5666E+01	0.6309E+04	0.6573E-01	0.7518E-09
1-10	0.5666E+01	0.6309E+04	0.6361E-01	0.6526E-09
1-11	0.9727E+01	0.3758E+04	0.1186E+00	0.5315E-08
1-12	0.8288E+01	0.3202E+04	0.1186E+00	0.4550E-08
1-13	0.5666E+01	0.6704E+04	0.6469E-01	0.6802E-09
1-14	0.8288E+01	0.3356E+04	0.1190E+00	0.3929E-08
1-15	0.8993E+01	0.3642E+04	0.1206E+00	0.4580E-08
1-16	0.5666E+01	0.7026E+04	0.6361E-01	0.6586E-09
1-17	0.4936E+01	0.6120E+04	0.6559E-01	0.4344E-09
2- 1	0.2072E+01	0.7449E+03	0.1052E+00	0.6307E-09
2- 2	0.2820E+01	0.1014E+04	0.1049E+00	0.1049E-08
2- 3	0.3238E+01	0.1164E+04	0.1069E+00	0.1117E-08
2- 4	0.3684E+01	0.1324E+04	0.1071E+00	0.1352E-08
2- 5	0.3917E+01	0.1408E+04	0.1060E+00	0.1570E-08
2- 6	0.4159E+01	0.1495E+04	0.1082E+00	0.1862E-08
2- 7	0.4407E+01	0.1584E+04	0.1080E+00	0.1913E-08
2- 8	0.4662E+01	0.1676E+04	0.1080E+00	0.2474E-08
2- 9	0.4159E+01	0.1607E+04	0.1036E+00	0.1470E-08
2-10	0.3917E+01	0.1514E+04	0.1051E+00	0.1381E-08
2-11	0.3684E+01	0.1423E+04	0.1066E+00	0.1260E-08
2-12	0.3457E+01	0.1336E+04	0.1026E+00	0.1205E-08
2-13	0.4159E+01	0.1684E+04	0.1061E+00	0.1242E-08
2-14	0.3917E+01	0.1586E+04	0.1051E+00	0.1211E-08
2-15	0.3684E+01	0.1492E+04	0.1043E+00	0.1042E-08
2-16	0.3238E+01	0.1458E+04	0.1010E+00	0.9280E-09
2-17	0.3684E+01	0.1659E+04	0.1043E+00	0.8815E-09
2-18	0.3917E+01	0.1764E+04	0.1051E+00	0.1000E-08
2-19	0.4159E+01	0.1873E+04	0.1036E+00	0.1184E-08
2-20	0.4662E+01	0.2099E+04	0.1080E+00	0.1365E-08

3- 1	0.3626E+01	0.3992E+04	0.6918E-01	0.4982E-09
3- 2	0.3935E+01	0.4332E+04	0.6840E-01	0.5230E-09
3- 3	0.4256E+01	0.4685E+04	0.6896E-01	0.5205E-09
3- 4	0.4589E+01	0.5052E+04	0.7014E-01	0.6491E-09
3- 5	0.4936E+01	0.5434E+04	0.6974E-01	0.6770E-09
3- 6	0.5294E+01	0.5829E+04	0.7038E-01	0.7410E-09
3- 7	0.5666E+01	0.6238E+04	0.6970E-01	0.9043E-09
3- 8	0.3935E+01	0.4656E+04	0.6840E-01	0.4550E-09
3- 9	0.4589E+01	0.5430E+04	0.6925E-01	0.6044E-09
3-10	0.4936E+01	0.5840E+04	0.6933E-01	0.6699E-09
3-11	0.5294E+01	0.6265E+04	0.6923E-01	0.7443E-09
3-12	0.5666E+01	0.6704E+04	0.6970E-01	0.8607E-09
3-13	0.4589E+01	0.5691E+04	0.6748E-01	0.5822E-09
3-14	0.4936E+01	0.6120E+04	0.6850E-01	0.6293E-09
3-15	0.5294E+01	0.6565E+04	0.6923E-01	0.7228E-09
3-16	0.5666E+01	0.7026E+04	0.6827E-01	0.8790E-09
3-17	0.4589E+01	0.6329E+04	0.6571E-01	0.5342E-09
3-18	0.4936E+01	0.6806E+04	0.6768E-01	0.6667E-09
3-19	0.5294E+01	0.7301E+04	0.6776E-01	0.7791E-09
3-20	0.5666E+01	0.7814E+04	0.6684E-01	0.8801E-09
3-21	0.4589E+01	0.6880E+04	0.6571E-01	0.5912E-09
3-22	0.4936E+01	0.7399E+04	0.6686E-01	0.6490E-09
3-23	0.5294E+01	0.7937E+04	0.6693E-01	0.7438E-09
3-24	0.5666E+01	0.8494E+04	0.6612E-01	0.8168E-09

D. COMPUTER PROGRAMS

```
//IGPCHJH JOB TIME=(3,00)
//  EXEC FORTRAN
C
C   METHOD OF BOX-SUPER BOX-MODIFIED OCT,1973, DR. STENSTROM
C   MODIFIED FOR OXYGEN TRANSFER MODEL WITH CHANGING KLA BY H.J. HWANG
C
C
C             DC
C             --    =  (A * R(T) + B) * (C* - C) - R(T)
C             DT
C
C             WHERE   R(T) = R(2) * EXP(-R(1) * T) + R(3)
C
C
C   SUBROUTINE SIGMA CONTROLS EVALUATION OF OBJECTIVE FUNCTION
C   FUNCTION IFIN DETERMINES IF THE FINISH CONDITION IS SATISFIED
C   ALPHA=RATIO  OF PROJECTION DISTANCE
C   CSTR=ARRAY CONTAINING CONSTRAINTS
C   ERROR=VARIABLE TO BE MINIMIZED
C   INDEX=NUMBER OF REJECTED POINT
C   ITERM=MAXIMUM NUMBER OF ITERATIONS BEFORE TERMINATION
C   N=NUMBER OF VARIABLES
C   M=NUMBER OF REQUIRED POINTS
C   MIN=MINIMUM ABSOLUTE ERROR FOR TERMINATION
C   MIN2=MINIMUM VARIANCE OF COST VALUES FOR TERMINATION
C   RAN=SEED FOR RANDOM NUMBER GENERATOR
C   VINTR=ARRAY CONTAINING CENTROID POINTS
C
      INTEGER RAN,TITLE(20)
      REAL MIN,MIN2
      DIMENSION INTS(57),REALS(1000),DATA3(2,100),RK(3),DELX(3)
      DIMENSION CFAC(5),YPRED(110),YRESID(110)
      DIMENSION VALS(4,30),CSTR(4,2),COST(30),SVALS(10),VINTR(10),
     1KEY(15),DATA1(100),DATA2(100),BEST(11)
      COMMON /NAME/ INTS,REALS
      EQUIVALENCE (REALS(1),VALS(1,1)),(REALS(301),COST(1)),(REALS(331),
     1SVALS(1)),(REALS(341),VINTR(1)),(REALS(351),CSTR(1,1)),(REALS(371)
     2,MIN),(REALS(372),ERROR),(REALS(373),Y),(REALS(374),WORST),
     3(REALS(375),ALPHA),(REALS(376),FACTOR),(REALS(377),MIN2),
     4(REALS(391),RK(1)),(REALS(601),YPRED(1)),(REALS(711),YRESID(1)),
     5(REALS(401),DATA1(1)),(REALS(501),DATA2(1)),(REALS(378),BEST(1)),
     6(REALS(821),DELX(1))
      EQUIVALENCE (INTS(16),N),(INTS(17),M),(INTS(18),
     1INDEX),(INTS(19),ITERM),(INTS(20),ITER),(INTS(21),RAN),
     2(INTS(23),NUM),(INTS(24),ISTUCK),
     3(INTS(37),TITLE(1)),(INTS(57),NSTEP)
C
      RAN=65467
      ALPHA=1.3
```

```
C.. SPECIFY THE FINISH CONDITION
C   JUMP4=1 FOR SIMPLE MINIMUM; JUMP4=2 FOR VARIANCE OF COST VALUE
C   JUMP4=3 FOR OTHER TYPES
      JUMP4=2
C.. SPECIFY THE NUMBER OF VARIABLES
      N=4
      M=N*2
      FACTOR=1./FLOAT(M-1)
C.. SPECIFY THE MAXIMUM NUMBER OF ITERATIONS
      ITERM=300
C.. SPECIFY THE MINIMUM ERROR
C
      MIN = 0.01
C
      MIN2=MIN**2
C
C.. WRITE THE HEADINGS
C
      WRITE(6,6100)
      WRITE (6,6110)
      WRITE(6,6210) N,M,MIN,MIN2,JUMP4,RAN
C
C.. READ THE TITLE AND INPUT DATA
C
 25   READ (5,5100, END=9999) (CSTR(IC,1),IC=1,4)
      READ (5,5100) (CSTR(IC,2), IC=1,4)
C
      READ (5,5100) (RK(I),I=1,3)
      READ(5,5200,END=9999) TITLE
C
      READ (5,5300) (CFAC(IF),IF=1,2),TSTART
      WRITE(6,6120) (TITLE(K),K=1,18)
      WRITE (6,6220) (RK(I),I=1,3)
C
      RK(2) = RK(2)*EXP(-RK(1)*TSTART)/60.
      RK(3) = RK(3)/60.
C
      WRITE (6,6230) TSTART
      WRITE (6,6220) (RK(I),I=1,3)
C
      DO 511 IR = 1,100
      READ (5,5100) DATA1(IR),(DATA3(IP,IR),IP=1,2)
      IF (DATA1(IR).LT.0.) GO TO 512
 511  CONTINUE
C
 512  NUM=IR-1
C
      RSTART=RK(2)+RK(3)
      REND=RK(2)*EXP(-RK(1)*DATA1(NUM))+RK(3)
      WRITE (6,6150) RSTART,REND
 6150 FORMAT (10X,'UPTAKE RATE CHANGES DURING TEST',/,
     1        20X,'FROM',F10.4,4X,'TO',F10.4,/)
C
```

```
C
C  C.. THIS PROGRAM IS FOR CONSTANT TIME INTERVAL
C
      NSTEP=10
      DELX(1)=(DATA1(2)-DATA1(1))/NSTEP
      DELX(2)=DELX(1)/2.
      DELX(3)=DELX(1)/6.
C
C.. START TO ESTIMATE PARAMETERS
C
      DO 315 IP=1,2
      INDEX=1
      ITER=1
      KEY(6)=0
      DO 30 IR=1,NUM
 30   DATA2(IR)=DATA3(IP,IR)*CFAC(IP)
C
      READ (5,5400) (VALS(I,INDEX),I=1,N)
C
      CALL SIGMA
C
      JUMP2=ICON(JUMP5)
      IF(JUMP2-1) 41,45,45
41    COST(INDEX)=ERROR
      IF(IP.EQ.1) WRITE(6,6130) IP
      IF(IP.GT.1) WRITE(6,6140) IP
C
      WRITE (6,6250)
      WRITE (6,6390)
      WRITE (6,6240) (CSTR(IC,1),IC=1,4)
      WRITE (6,6240) (CSTR(IC,2),IC=1,4)
C
      WRITE(6,6310)
C     WRITE(6,6250)
      WRITE(6,6240) (VALS(I,INDEX),I=1,N),COST(INDEX)
      GO TO 55
45    WRITE(6,6320)
      INDEX=0
55    WRITE(6,6330)
C     WRITE(6,6250)
60    INDEX=INDEX+1
70    IF(KEY(6).LT.300) GO TO 75
      WRITE(6,6340) KEY(6)
      GO TO 315
75    KEY(6)=KEY(6)+1
C
C.. CALCULATE M-1 STARTING POINTS
C
      DO 100 J=1,N
      IY=RAN*65539
      IF(IY) 80,80,90
80    IY=IY+2147483647 +1
90    Y=IY
```

```
      Y=Y*0.4656613D-09
      VALS(J,INDEX)=(CSTR(J,2)-CSTR(J,1))*Y+CSTR(J,1)
      RAN=IY
 100  RAN=IFIX(FLOAT(RAN)*Y)
      JUMP2=ICON(JUMP5)
      IF(JUMP2.EQ.1) GO TO 70
      CALL SIGMA
      JUMP2=ICON(JUMP5)
      IF(JUMP2.EQ.1) GO TO 70
      KEY(6)=0
      COST(INDEX)=ERROR
      WRITE(6,6240) (VALS(I,INDEX),I=1,N),COST(INDEX)
      IF(M-INDEX) 140,140,60
 120  ITER=ITER+1
      COST(INDEX)=ERROR
C
C.. CHECK FOR TERMINAL CONDITIONS
C
      JUMP3=IFIN(JUMP4)
      GO TO (130,230),JUMP3
C
C..  CHECK ITERATION NUMBER
C
130   IF(ITER.GE.ITERM) GO TO 220
C
C.. FIND WORST VALUE
C
140   WORST=COST(1)
      INDEX=1
      DO 150 I=2,M
      IF(WORST.GT.COST(I)) GO TO 150
      WORST=COST(I)
      INDEX=I
150   CONTINUE
C
C.. INITILIZE ARRAY AND CALCULATE NEW VALS
C
      ISTUCK=0
      DO 160 I=1,N
160   SVALS(I)=0.0
      DO 170 I=1,N
      DO 170 J=1,M
170   SVALS(I)=SVALS(I)+VALS(I,J)
      DO 180 I=1,N
      VINTR(I)=FACTOR*(SVALS(I)-VALS(I,INDEX))
180   VALS(I,INDEX)=ALPHA*(VINTR(I)-VALS(I,INDEX))+VINTR(I)
C
C.. CHECK TO SEE IF CONSTRAINTS ARE VIOLATED
C
      JUMP2=ICON(JUMP5)
      IF(JUMP2.EQ.1) GO TO 200
      CALL SIGMA
      JUMP2=ICON(JUMP5)
```

```
      IF(JUMP2.EQ.1) GO TO 200
190   IF(ERROR.LE.COST(INDEX)) GO TO 120
C
C.. NO IMPROVEMENT-MOVE HALFWAY BACK TO THE CENTROID
C
      ITER=ITER+1
200   DO 210 I=1,N
210   VALS(I,INDEX)=0.50*(VALS(I,INDEX)+VINTR(I))
      ISTUCK=ISTUCK+1
      IF(ISTUCK-10) 215,215,216
215   JUMP2=ICON(JUMP5)
      IF(JUMP2.EQ.1) GO TO 200
C
      CALL SIGMA
C
      JUMP2=ICON(JUMP5)
      IF(JUMP2.EQ.1) GOTO 200
      IF(ITER.GE.ITERM) GO TO 220
      GO TO 190
216   WRITE(6,6350) ISTUCK
      GO TO 240
220   WRITE(6,6360)
      GO TO 240
230   WRITE(6,6370)
C240   WRITE(6,6250)
 240  CONTINUE
      DO 250 I=1,M
250   WRITE(6,6240) (VALS(J,I),J=1,N),COST(I)
      WRITE(6,6380) ITER
C
C.. FIND THE LOWEST VALUE OF COST AND PRINT OUT THE VALS
C
970   INDEX = 1
      DO 260 I=2,M
260   IF(COST(INDEX).GT.COST(I)) INDEX=I
      WRITE(6,6260) INDEX
C      WRITE(6,6250)
      WRITE(6,6240) (VALS(I,INDEX),I=1,N),COST(INDEX)
C
C.. SAVE THE BEST VALUES OF A SERIES OF RUNS
C
      DO 270 I=1,N
270   BEST(I)=VALS(I,INDEX)
      BEST(11)=COST(INDEX)
C
C.. WRITE OUT THE RESIDUALS, PREDICTED VALUES, AND CALCULATE THE STATIST
C
      WRITE(6,6270)
C
      CALL SIGMA
C
      WRITE (6,6280) (I,DATA1(I),DATA2(I),YPRED(I),YRESID(I),I=1,NUM)
C
```

```
C.. FORMATS
C
 5100 FORMAT (8F10.0)
 5200 FORMAT(20A4)
 5300 FORMAT (10X,7F10.0)
 5400 FORMAT (/,8F10.0)
 6100 FORMAT('1 MULTIVARIABLE OPTIMIZATION TECHNIQUE-SUPERBOX OCT-1973')
 6110 FORMAT ('0',///,9X,'*** VARIABLE KLA MODEL ***',/)
 6120 FORMAT(1H1,1X,18A4,/)
 6130 FORMAT ('-PROBE',I3,/)
 6140 FORMAT ('1PROBE',I3,/)
 6210 FORMAT('-NUMBER OF VARIABLES=',11X,I3,/,' NUMBER OF VERTICIES =',
     110X,I3,/,' MINIMUM ABSOLUTE ERROR= ',  6X,F10.5,/,' MINIMUM VARIAN
     2CE OF ERROR =',1X,F15.8,/,' FINISH CRITERIA =',15X,I2,/,' RAN =',
     419X,I10)
 6220 FORMAT (10X,'KU = ',F8.5,5X,'RO = ',F8.4,5X,'RE = ',F8.4,/)
 6230 FORMAT (10X,'TSTART = ', F7.2,/)
 6240 FORMAT(3X,4E16.5,7X,E16.5)
 6250 FORMAT(13X,2HK1,14X,2HK2,14X,2HK3,14X,2HK4,19X,4HCOST,/)
 6260 FORMAT (' THE VALUE OF INDEX FOR LOWEST COST = ',I2,/,' THE VALS
     1ARE:',/)
 6270 FORMAT('1VARIABLE KLA MODEL ... PREDICTIONS AND RESIDUALS',//,
     1' POINT NO.',5X,'TIME'
     2,5X,'DATA POINT',5X,'PREDICTED VALUE',14X,'RESIDUAL',/)
 6280 FORMAT (6X,I4,4X,F5.2,10X,F5.2,15X,F5.2,5X,E17.6)
 6310 FORMAT('-USER SUPPLIED STARTING POINT',/)
 6320 FORMAT(' THE USER SUPPLIED STARTING POINT DOES NOT SATISFY THE CON
     1STRAINTS.  A RANDOM POINT WILL BE SUBSTITUTED.',/)
 6330 FORMAT('0RANDOMLY GENERATED POINTS',/)
 6340 FORMAT(1X,I4,' UNSUCCESSFUL ATTEMPTS HAVE BEEN MADE TO OBTAIN A
     1RANDOM STARTING POINT. OPTIMIZATION TERMINATING',/,' NEXT DATA SET
     2WILL BE RUN')
 6350 FORMAT(' THE OPTIMIZATION IS APPARENTLY STUCK.  ISTUCK =',I3,/)
 6360 FORMAT(1H-,' THE MAXIMUM NUMBER OF ITERATIONS HAS BEEN EXCEEDED',
     1/)
 6370 FORMAT(1H-,'THE OPTIMIZATION HAS TERMINATED NORMALLY',/)
 6380 FORMAT(1H0,16HITERATION COUNT=,I5)
 6390 FORMAT(' USER SUPPLIED LOWER AND UPPER CONSTRAINTS :',/)
C
 9999 STOP
      END
      BLOCK DATA
      DIMENSION INTS(57),REALS(1000)
      COMMON /NAME/ INTS,REALS
      DATA INTS/56*0/,REALS/1000*0./
      END
C
C
C
      SUBROUTINE SIGMA
C
C.. SUBROUTINE SIGMA CONTROLS EVALUATION OF OBJECTIVE FUNCTION
C.. USING FOURTH ORDER RUNGE-KUTTA NUMERICAL INTEGRATION
```

```
C
C    NUM = NUMBER OF DATA POINTS TO BE READ (IF ANY)
      INTEGER RAN
      REAL MIN,MIN2
      DIMENSION INTS(57),REALS(1000)
      DIMENSION VALS(4,30),CSTR(4,2),COST(30),SVALS(10),VINTR(10),
     1KEY(15),DATA1(100),DATA2(100)
      DIMENSION RK(3),DELX(3),YPRED(110),YRESID(110)
      COMMON /NAME/ INTS,REALS
      EQUIVALENCE (REALS(1),VALS(1,1)),(REALS(301),COST(1)),(REALS(331),
     1SVALS(1)),(REALS(341),VINTR(1)),(REALS(351),CSTR(1,1)),(REALS(371)
     2,MIN),(REALS(372),ERROR),(REALS(373),Y),(REALS(374),WORST),
     3(REALS(375),ALPHA),(REALS(376),FACTOR),(REALS(377),MIN2),
     4(REALS(391),RK(1)),(REALS(601),YPRED(1)),(REALS(711),YRESID(1)),
     5(REALS(401),DATA1(1)),(REALS(501),DATA2(1)),
     6(REALS(821),DELX(1))
      EQUIVALENCE (INTS(16),N),(INTS(17),M),(INTS(18),
     1INDEX),(INTS(19),ITERM),(INTS(20),ITER),(INTS(21),RAN),
     2(INTS(23),NUM),(INTS(57),NSTEP)
C
      YPRED(1)=VALS(4,INDEX)
      YRESID(1)=-YPRED(1)+DATA2(1)
      ERROR=YRESID(1)**2
      XOLD=DATA1(1)
      YOLD=VALS(4,INDEX)
      UPTAKE=RK(2)*EXP(-RK(1)*XOLD)+RK(3)
C
      DO 50 I=2,NUM
C
      XNEW=DATA1(I-1)
C
      DO 100 IS=1,NSTEP
      XNEW=XOLD+DELX(1)
      XHALF=XOLD+DELX(2)
      ZK1=(VALS(1,INDEX)*UPTAKE+VALS(2,INDEX))*(VALS(3,INDEX)-YOLD)-
     1    UPTAKE
C
      YMID=YOLD+DELX(2)*ZK1
      UPTAKE=RK(2)*EXP(-RK(1)*XHALF)+RK(3)
      ZK2=(VALS(1,INDEX)*UPTAKE+VALS(2,INDEX))*(VALS(3,INDEX)-YMID)-
     1    UPTAKE
C
      YMID=YOLD+DELX(2)*ZK2
      ZK3=(VALS(1,INDEX)*UPTAKE+VALS(2,INDEX))*(VALS(3,INDEX)-YMID)-
     1    UPTAKE
C
      YMID=YOLD+DELX(1)*ZK3
      UPTAKE=RK(2)*EXP(-RK(1)*XNEW)+RK(3)
      ZK4=(VALS(1,INDEX)*UPTAKE+VALS(2,INDEX))*(VALS(3,INDEX)-YMID)-
     1    UPTAKE
C
      YOLD=YOLD+DELX(3)*(ZK1+2.*ZK2+2.*ZK3+ZK4)
 100  XOLD=XNEW
```

```
      YPRED(I)=YOLD
      YRESID(I)=-YPRED(I)+DATA2(I)
 50   ERROR=ERROR+YRESID(I)**2
C
      RETURN
      END
C
      FUNCTION ICON(JUMP5)
C.. FUNCTION ICON DETERMINES IF A CONSTRAINT VIOLATION HAS OCCURRED
C   ICON=1 IF THE CONSTRAINT HAS BEEN VIOLATED
      DIMENSION INTS(57),REALS(1000)
      DIMENSION VALS(4,30),CSTR(4,2),KEY(15)
      COMMON /NAME/ INTS,REALS
      EQUIVALENCE (REALS(1),VALS(1,1)),(REALS(351),CSTR(1,1))
      EQUIVALENCE (INTS(16),N),(INTS(18),INDEX)
      ICON=0
      DO 10 I=1,N
      IF(VALS(I,INDEX).LT.CSTR(I,1)) ICON=1
10    IF(VALS(I,INDEX).GT.CSTR(I,2)) ICON=1
C                                          IF(KEY(5).GE.1) ICON=1
C                                                 KEY(5)=0
      RETURN
      END
C
C
C
      FUNCTION IFIN(JUMP4)
C.. FUNCTION IFIN DETERMINES IF THE FINISH CONDITION IS SATISFIED
C   IFIN=2 IF THE FINISH CRITERIA HAS BEEN SATISIFIED
C   IFIN =1 OTHERWISE
      REAL MIN,MIN2
      DIMENSION INTS(57),REALS(1000)
      DIMENSION COST(30),KEY(15)
      COMMON /NAME/ INTS,REALS
      EQUIVALENCE (REALS(301),COST(1)),(REALS(371),MIN)
     1,(INTS(17),M),(INTS(18),INDEX),(REALS(377),MIN2)
      GO TO (100,200,300),JUMP4
100   IF(COST(INDEX).LT.MIN) GO TO 120
      IFIN=1
      RETURN
120   IFIN=2
      RETURN
200   S1=0.
      S2=0.
      DO 220 I=1,M
      S1=S1+COST(I)
220   S2=S2+COST(I)**2
      V=(S2-S1**2/FLOAT(M))/FLOAT(M-1)
      IF(V.LT.MIN2) GO TO 230
      IFIN=1
      RETURN
230   IFIN=2
      RETURN
```

```
300   CONTINUE
C.. USER SUPPLIED FINISH CONDITION
      IFIN=1
      RETURN
      END
//GO.SYSIN DD *
  -0.13      0.22       8.8        1.5
  -0.08      0.27       9.3        1.9
  0.01550    15.3037    17.4564
 ALPHA BETA TEST RUN 65 ... 12/07/81
             1.8299     1.8481     62.7
  0.         0.92       1.00
  1.         1.55       1.56
  2.         1.97       1.99
  3.         2.34       2.37
  4.         2.65       2.65
  5.         2.86       2.85
  6.         3.08       3.05
  7.         3.22       3.23
  8.         3.40       3.35
  9.         3.50       3.45
  10.        3.63       3.54
  11.        3.65       3.61
  12.        3.75       3.68
  13.        3.77       3.72
  14.        3.82       3.73
  15.        3.85       3.81
  16.        3.89       3.82
  17.        3.93       3.85
  18.        3.95       3.90
  19.        3.95       3.90
  20.        3.96       3.92
  21.        3.98       3.90
  22.        4.00       3.92
  23.        4.00       3.94
  24.        4.00       3.95
   -1.
 AB TEST RUN 65, ML, 14.2FT, G43 ... PROBE 1
  -0.1       0.24       9.0
 AB TEST RUN 65, ML, 14.2FT, G43 ... PROBE 2
  .0
//
```

```
//IGPCHJH JOB 'HYUNG,J,HWANG',TIME=(,60)
//  EXEC VERSATEC
C.......... PROGRAM FOR SUBROUTINE DOH 7/18/81, HWANG
C.......... MAX. NUMBER OF DATA SET IS 6(=NP)
C.......... INPUT FORMAT
C..........     UPTAKE PARAMETERS................. (8F10.0)
C..........     TITLE ............................ (8X,12A4)
C..........     CONVERSION FACTOR, STARTING TIME.. (10X,7F10.0)
C..........     T(J),(CMV(I,J),I=1,NP) ........... (8F10.0)
C..........     NEGATIVE VALUE AFTER T.VS.CMV .... (F10.0)
C..........     KLA,CS,CO......................... (3F10.0)
C.......... INITIAL ESTIMATES ARE REQUIRED FOR EVERY T.VS.CMV DATA
C..........
      REAL VAR(3),CMV(6,100),CONV(6)
      COMMON /GROUP1/T(100),COBS(100),PARM(3),R(3),IDEBUG,INP
      COMMON /GROUP2/ITITLE(12)
      DATA R/3*0./
      NP=2
C
C.........READ PARAMETERS FOR UPTAKE
C
 10   READ (5,5200,END=9999) (R(I),I=1,3)
C
      READ (5,5000,END=9999) (ITITLE(I),I=1,12)
      WRITE (6,6100) (ITITLE(I),I=1,12)
      WRITE (6,6200) (R(I),I=1,3)
C
      R(3)=R(3)/60.
      READ (5,5100)  (CONV(I),I=1,NP),TSET
      DO 100 J=1,100
      READ (5,5200) T(J),(CMV(I,J),I=1,NP)
      IF (T(J).LT.0.0) GO TO 110
 100  CONTINUE
 110  TINIT=T(1)
      WRITE (6,6300) TSET,TINIT
      N=J-1
      TSET=TINIT+TSET
      R(2)=R(2)*EXP(-R(1)*TSET)/60.
      WRITE (6,6200) (R(IT),IT=1,3)
      DO 200 J=1,N
 200  T(J)=T(J)-TINIT
        DO 300 INP=1,NP
          DO 310 J=1,N
 310      COBS(J)=CMV(INP,J)*CONV(INP)
          READ (5,5000) (ITITLE(I),I=1,12)
          READ (5,5300) (VAR(I),I=1,3),IDEBUG
          IF (VAR(1)) 320,350,330
 320      CALL GUESS (N)
          GO TO 350
 330         DO 340 I=1,2
 340         PARM(I)=VAR(I)
 350      PARM(3)=COBS(1)
          CALL DOH(N)
```

```
  300  CONTINUE
       GO TO 10
 9999  WRITE (6,6400)
 5000  FORMAT (12A4)
 5100  FORMAT (10X,7F10.0)
 5200  FORMAT (8F10.0)
 5300  FORMAT (3F10.0,I1)
 6100  FORMAT (1H1,3X,12A4)
 6200  FORMAT (/,5X,'UPTAKE RATE PARAMETERS ARE',/,
     1         /,10X,'R(1)=',F9.5,5X,'R(2)=',F9.4,5X,'R(3)=',F9.4)
 6300  FORMAT (/,5X,'STARTING TIME = ',F6.1,
     1         /,5X,'INITIAL TIME = ',F6.1)
 6400  FORMAT (1H1,'    *** END OF DATA ***')
       STOP
       END
C
C
C
       SUBROUTINE GUESS (ND)
C
C.......THIS PROGRAM ESTIMATE STARTING VALUES OF PARAMETERS
C.......FOR TAP WATER TEST DATA
C
       COMMON /GROUP1/T(100),COBS(100),PARM(3),R(3),IDEBUG,INP
       PARM(2)=COBS(ND)/.98
       PARM(3)=COBS(1)
       CKLA1=PARM(2)*.86
       DO 10 I=1,ND
       IF (COBS(I).GT.CKLA1) GO TO 20
  10   CONTINUE
  20   IF ((CKLA1-COBS(I-1)).GT.(COBS(I)-CKLA1)) GO TO 30
       I=I-1
  30   PARM(1)=-ALOG((PARM(2)-COBS(I))/(PARM(2)-PARM(3)))/T(I)
       RETURN
       END
C
C
C
       SUBROUTINE DOH (ND)
C.........OXYGEN TRANSFER PARAMETER ESTIMATION
C.........JULY 7,1981    HYUNG J.HWANG
C
C.........THIS PROGRAM CAN ESTIMATE PARAMETERS FOR FOLLOWING EQUATIONS
C
C...CLEAN WATER DATA
C
C          C  =  C* - (C* - C ) EXP(-K A * T)
C                            0        L
C
C...DIRTY WATER DATA
C
C.........CONSTANT UPTAKE RATE
C
```

```
C         C  =  C  - (C  - C ) EXP(-K A * T)
C                R     R    0       L
C
C.........EXPONENTIAL DECAY OUR, BUT CONSTANT K A
C                                              L
C
C         C = C  - (C  - F - C ) * EXP(-K A * T)- F*EXP(-R(1)*T)
C              R     R        0          L
C
C                                R(3)               R(2)
C               WHERE  C   = C* - ----   AND  F =  ----------
C                       R         K A              K A - R(1)
C                                  L                 L
C
      REAL CHAT(120),RESID(100),Z(3),A(3,4),DELP(3),
     1     SIGP(3),TD(120)
      COMMON /GROUP1/T(100),COBS(100),PARM(3),R(3),IDEBUG,INP
      COMMON /GROUP2/ITITLE(12)
      DATA EPS/1.E-5/
      U3=1.
      U4=1.
      IF (INP.EQ.1) WRITE (6,6100) (ITITLE(K),K=1,12)
      IF (INP.GT.1) WRITE (6,6101) (ITITLE(K),K=1,12)
      WRITE (6,6110) (PARM(K),K=1,3)
C
C..............DEBUGGING
          IF (IDEBUG.EQ.0) GO TO 1
          WRITE (6,6120) (IP,IP=1,3)
 1        CONTINUE
C
      ITER=0
C
 1000 ITER=ITER+1
      RMAX=0.0
      DO 10 IP=1,3
      DO 10 JP=1,4
 10   A(IP,JP)=0.0
C
C..........EVALUATE COEFFICIENT MATRIX
C
C
         G1=R(2)/(PARM(1)-R(1))
         G2=G1/(PARM(1)-R(1))
         G3=R(3)/PARM(1)
         G4=G3/PARM(1)
C
      DIFF=PARM(2)-PARM(3)
        DO 15 J=1,ND
        CHAT(J)=PARM(2)-(PARM(2)-PARM(3)-G1-G3)*EXP(-PARM(1)*T(J))-
     1          G1*EXP(-R(1)*T(J))-G3
        RESID(J)=COBS(J)-CHAT(J)
 15     IF (RMAX.LT.ABS(RESID(J))) RMAX=ABS(RESID(J))
          DO 30 J=1,ND
```

```
          Z(3)=EXP(-PARM(1)*T(J))
          Z(2)=1.-Z(3)
C
          Z(1)=(-G2-G4+T(J)*((PARM(2)-PARM(3))-G1-G3))*Z(3)
     1         +G2*EXP(-R(1)*T(J))+G4
C
          DO 30 IP=1,3
          DO 20 JP=IP,3
 20       A(IP,JP)=A(IP,JP)+Z(IP)*Z(JP)
 30       A(IP,4)=A(IP,4)+Z(IP)*RESID(J)
C
C..........REVISE THE COEFFICIENT MATRIX
C
      ABAR=A(2,2)*A(3,3)-A(2,3)**2
        DO 40 IP=2,3
        DO 40 JP=IP,4
 40     A(IP,JP)=A(1,1)*A(IP,JP)-A(1,IP)*A(1,JP)
C
C..........EVALUATE CHANGES OF PARAMETERS
C
      DELP(3)=(A(2,2)*A(3,4)-A(2,3)*A(2,4))/(A(2,2)*A(3,3)-A(2,3)**2)
      DELP(2)=(A(2,4)-A(2,3)*DELP(3))/A(2,2)
      DELP(1)=(A(1,4)-A(1,2)*DELP(2)-A(1,3)*DELP(3))/A(1,1)
C
C.........DEBUGGING
C
         IF (IDEBUG.EQ.0) GO TO 45
         WRITE (6,6130) ITER,(DELP(IP),IP=1,3)
 45      CONTINUE
C
C..........UPDATE PARAMETERS
C
        DO 50 IP=1,3
 50     PARM(IP)=PARM(IP)+DELP(IP)
C
C..........CHECK CONVERGENCE
C
        DO 60 IP=1,3
        IF (ABS(DELP(IP)/PARM(IP)).GT.EPS ) GO TO 70
 60     CONTINUE
      GO TO 80
 70   IF (ITER.LT.15) GO TO 1000
      WRITE (6,6200)
 80   DET=(A(2,2)*A(3,3)-A(2,3)**2)/A(1,1)
C
      WRITE (6,6300) ITER
      WRITE (6,6400) PARM(1),DELP(1),PARM(2),DELP(2),PARM(3),DELP(3)
C
C..........EVALUATE RESIDUALS
C
      WRITE (6,6500)
      RS=0.0
      RSS=0.0
```

```
C
          G1=R(2)/(PARM(1)-R(1))
          G3=R(3)/PARM(1)
C
        DO 90 J=1,ND
        CHAT(J)=PARM(2)-(PARM(2)-PARM(3)-G1-G3)*EXP(-PARM(1)*T(J))-
     1           G1*EXP(-R(1)*T(J))-G3
C       CHAT(J)=PARM(2)-(PARM(2)-PARM(3))*EXP(-PARM(1)*T(J))
        RESID(J)=COBS(J)-CHAT(J)
        WRITE (6,6600) T(J),COBS(J),CHAT(J),RESID(J)
        RS=RS+RESID(J)
 90     RSS=RSS+RESID(J)**2
      RMEAN=RS/ND
      RSIGMA=(RSS-ND*RMEAN**2)/(ND-1)
      RMS=RSS/(ND-3)
C
C
C...........STANDARD ERRORS OF ESTIMATED PARAMETERS
C
      A(1,1)=ABAR
      WRITE (6,6700)
        DO 100 IP=1,3
 100    SIGP(IP)=A(IP,IP)*RMS/DET
      WRITE (6,6800) RS,RSS,RMEAN,RSIGMA,RMS,(SIGP(IP),IP=1,3)
C
 6100 FORMAT (//,15X,5('+----'),'+',///,4X,12A4)
 6101 FORMAT (1H1,3X,12A4)
 6110 FORMAT (//,5X,'INITIAL ESTIMATE OF PARAMETERS',
     1        //,10X,'KLA = ',F5.2,5X,'C* = ',F5.2,5X,'CO = ',F5.2,/)
 6120 FORMAT (/,11X,'ITER',3(6X,'DELP(',I1,')',2X),/)
 6130 FORMAT (13X,I2,3E15.4)
 6200 FORMAT (/,5X,'--- MAX ITERATION NUMBER IS REACHED ---',/)
 6300 FORMAT (/,5X,'NUMBER OF ITERATION = ',I3)
 6400 FORMAT (//, 5X,'ESTIMATED VALUES OF PARAMETERS',
     1        //,10X,'... KLA = ',F10.6,10X,'DKLA = ',E15.6,
     2        //,10X,'... C*  = ',F10.6,10X,'DCS  = ',E15.6,
     3        //,10X,'... CO  = ',F10.6,10X,'DCO  = ',E15.6)
 6500 FORMAT (1H1,10X,'TIME',4X,'OBSERVED   PREDICTED    RESIDUAL',/)
 6600 FORMAT (8X,F7.2,2F12.5,F12.6,F12.5)
 6700 FORMAT (//,10X,'***** STATISTICS *****',/)
 6800 FORMAT (10X,'RESIDUAL SUM     = ',E15.6,/,
     1        10X,'RESIDUAL SUM OF SQUARES  = ',E15.6,/,
     2        10X,'RESIDUAL MEAN            = ',E15.6,/,
     3        10X,'RESIDUAL STD. DEV.       = ',E15.6,/,
     4        10X,'RESIDUAL MEAN OF SQUARES = ',E15.6,//,
     5        10X,'SIGMA KLA = ',E15.6,/,
     6        10X,'SIGMA CS  = ',E15.6,/,
     7        10X,'SIGMA CO  = ',E15.6)
       DELT=T(ND)/100.
       DO 200 J=1,100
       TD(J)=(J-1)*DELT
 200   CHAT(J)=PARM(2)-(PARM(2)-PARM(3)-G1-G3)*EXP(-PARM(1)*TD(J))-
     1          G1*EXP(-R(1)*TD(J))-G3
```

```
C200    CHAT(J)=PARM(2)-(PARM(2)-PARM(3))*EXP(-PARM(1)*TD(J))
C
C...................... DEBUG START
            IPLOT=1
            IF (IPLOT.EQ.1) RETURN
C...................... DEBUG END
C
          CALL CPLOT(3,1.,ND,-3,T,COBS,1,1,IPOS)
          CALL CPLOT(3,1.,100,1,TD,CHAT,1,1,IPOS)
          CALL CPLOT(3,1.,100,5,TD,CHAT,1,1,IPOS)
          CALL CPLOT(3,1.,ND,-4,T,RESID,1,3,IPOS)
          CALL CPLOT(3,1.,ND, 5,T,RESID,1,3,IPOS)
      RETURN
      END
```

PROGRAM TO ANLYYZE AERATION TEST DATA

```
//IGPCHJH JOB 'HYUNG,J,HWANG',TIME=(0,30)
//  EXEC VERSATEC
C.....KLAT(IP,IK)
C        IP=1....PROBE 1        IK=1....SLOPR
C           2....      2           2....KLAT FOR CONSTANT KLA, OR
C           3....MEAN VALUE             KLAT AT R=1 MG/L/MIN
C           4....% VARIATION
C.....KLAS(IK)
C        IK=1....SLOPE
C           2....KLAS FOR CONSTANT KLA, OR
C                KLAS AT R=1 MG/L/MIN
C           3....KLAS AT START (UTRS)
C           4....KLAS AT END   (UTRE)
C.....CST(IP)
C        IP=1....PROBE 1
C           2....      2
C           3....MEAN VALUE
C           4....% VARIATION
C.....SOTR(IS)
C              CONSTANT KLA            VARIABLE KLA
C        IS=1....         -----      SOTR AT START (G/MIN)
C           2....SOTR     -----        "     END   (  "  )
C           3....         -----      SOTR AT START (IB/HR/1000CU-FT
C           4....SOTR     -----        "     END   (IB/HR/1000CU-FT)
C.....OTE(IO)
C        IO=1....OTE OR OTE AT START
C           2....        "      END
C.....TEMP(I)     I=1....WATER (C)
C                   2....COMP AIR (F)
C                   3....AMB AIR (C)
C                   4....INLET AIR (C)
C.....PRES(I)     I=1....BAROMETRIC (MM-HG)
C                   2....MERCURY MANOMETER (IN-HG)
C                   3....PRESSURE DROP, ORIFICE (IN-H2O)
C                   4....PRESSURE DROP, ORIFICE AND DIFFUSER (IN-H2O)
C                   5....VAPOR PRESSURE AT WATER TEMP. (MM-HG)
C                   6....    "       "   AT INLET AIR TEMP. (MM-HG)
C.....IUP = 1 ... ZERO MICROBIAL OXYGEN UPTAKE RATE (TAP WATER)
C          2 ... CONSTANT "      "      "     "
C          3 ... EXPONENTIAL DECREASE OF "   "   "   "
C
      REAL KLAT(8,8),KLAS(4),KLASV(4),KLA20(4),CST(4),TEMP(4),
     1  PRES(6),OTE(4),SOTR(8),RK(3)
C.. DIMENSION FOR PLOT
      REAL KLAP(20,30,2),CSSP(20,30),OTEP(20,30,2),SOTRP(20,30,2),
     1    QAIRP(20,30),UTRP(20,30,2),XT(30),YT(30),
     2    XP(5,15),YP(5,15),QP(7),
     3    QAIR(140),QAIRC(140),QAIRS(140),FP(140),PRESS(4,140)
C
      INTEGER IRUN(200),NC(40),NT(6),ICP(5)
```

```
      COMMON ITITLE(18)
      DATA NC/30*0/,WF,RHO,RHOF,DF,VMU/2.715,0.0012,2.53,0.5,0.0181/
C     ICMIN=100
C     ICMAX=1
      VFACT=20.**2*3.1415927/(4.*144.)
      CFACT=1000.*60./453.6
      OFACT=0.231*0.07521*0.9917*453.6
C
      DO 3000 JC=1,200
C
C....... READ IN DATA
      READ (5,5100) IRUN(JC),((KLAT(IP,IK),IK=1,2),IP=1,2),
     1              (RK(IR),IR=1,3),TS,TE,
     2              (CST(IP),IP=1,2),(TEMP(IT),IT=1,4),WDEP,
     3              QAIR(JC),(PRES(I),I=1,6),IC
      IF (IRUN(JC).LT.1) GO TO 4000
      NC(IC)=NC(IC)+1
C
      WRITE (6,6000) IRUN(JC)
      IUP=1
      IF(RK(3).NE.0.) IUP=2
      IF(RK(2).NE.0.) IUP=3
C
      IF (KLAT(2,2).EQ.0.) GO TO 10
C
C....... CALCULATE MEAN VALUE AND % VARIATION
      DO 5 IK=1,2
          IF (KLAT(1,IK).EQ.0.0) GO TO 5
          IF (KLAT(2,IK).EQ.0.0) GO TO 5
      KLAT(3,IK)=(KLAT(1,IK)+KLAT(2,IK))/2.
      KLAT(4,IK)=(KLAT(1,IK)-KLAT(3,IK))/KLAT(3,IK)*100.
 5    CONTINUE
      CST(3)=(CST(1)+CST(2))/2.
      CST(4)=(CST(1)-CST(3))/CST(3)*100.
      GO TO 20
C
 10   DO 15 IK=1,2
      KLAT(3,IK)=KLAT(1,IK)
 15   KLAT(4,IK)=0.
      CST(3)=CST(1)
      CST(4)=0.
C
C....... CLACULATE VOLUME OF WATER IN (CU-FT)
 20   VW=VFACT*WDEP
C
C....... C* VALUE IN BOOK
      CBOOK=CINTR(TEMP(1))
C
C....... EFFECTIVE DEPTH
      DE=(CST(3)/CBOOK*(760.-PRES(5))-PRES(1)+PRES(5))/22.42
C
C....... SATURATED CONCENTRATION AT 20 C
      CS20=CST(3)*9.07/CBOOK*(760.+22.42*DE-17.535)/(PRES(1)+
```

```
     1          22.42*DE-17.535)
C
C.....   CORRECTED AIR FLOW RATE
C
      FP(JC)=(PRES(1)/760.+PRES(2)/29.92)*293.15/(273.15+TEMP(2))
      QAIRC(JC)=QAIR(JC)*SQRT(FP(JC))
      FWV=1.0084*(1.-0.98*PRES(6)/PRES(1))
     1          /(1.-0.3775*0.98*PRES(6)/PRES(1))
      QAIRS(JC)=FWV*QAIRC(JC)
          PRESS(1,JC)=PRES(3)
          PRESS(2,JC)=PRESS(1,JC)*FP(JC)
          PRESS(3,JC)=PRES(4)-PRES(3)
          PRESS(4,JC)=PRESS(3,JC)*FP(JC)
C
C.....   OXYGEN SUPPLIED
C
      O2SUP=OFACT*QAIRS(JC)
C
      GO TO (1000,1000,2000),IUP
 1000 KLAS(2)=KLAT(3,2)*1.024**(20.-TEMP(1))
      SOTR(2)=KLAS(2)*CS20*VW*0.02832
      SOTR(4)=SOTR(2)*CFACT/VW
C
      OTE(2)=SOTR(2)/O2SUP*100.
C
      IF (IUP.EQ.2) WRITE (6,6110) RK(3)
      WRITE(6,6200) (KLAT(IP,2),IP=1,4)
      WRITE(6,6300) (CST(IP),IP=1,4),(TEMP(IT),PRES(IT),IT=1,4),
     1              PRES(5),WDEP,PRES(6),QAIR(JC),VW
C
      UTRS=RK(3)/60.
C
      WRITE(6,6400) DE,QAIRS(JC)
      WRITE(6,6600) CS20,CBOOK
      IF (IUP.EQ.1) WRITE (6,6500) KLAS(2)
      IF (IUP.EQ.2) WRITE (6,6510) KLAS(2),UTRS
      WRITE(6,6700) SOTR(2),OTE(2),SOTR(4)
C
C.. PREPARE TO PLOT
        QAIRP(IC,NC(IC))=QAIRS(JC)
        KLAP(IC,NC(IC),1)=KLAS(2)
        KLAP(IC,NC(IC),2)=0.0
        CSSP(IC,NC(IC))=CS20
        OTEP(IC,NC(IC),1)=OTE(2)
        SOTRP(IC,NC(IC),1)=SOTR(4)
        UTRP(IC,NC(IC),1)=UTRS
C
      GO TO 3000
 2000 UTRS=(RK(2)*EXP(-RK(1)*TS)+RK(3))/60.
      UTRE=(RK(2)*EXP(-RK(1)*(TS+TE))+RK(3))/60.
      DO 2010 IK=1,2
 2010 KLAS(IK)=KLAT(3,IK)*1.024**(20.-TEMP(1))
C
```

```
C........ KLAS DURING EXPERIMENT
      KLAS(3)=KLAS(1)*UTRS+KLAS(2)
      KLAS(4)=KLAS(1)*UTRE+KLAS(2)
C
C...... CALCULATE STANDART OXYGEN TRANSFER RATE AND EFFICIENCY
      DO 2020 IK=1,2
      SOTR(IK)=KLAS(IK+2)*CS20*VW*0.02832
      SOTR(IK+2)=SOTR(IK)*CFACT/VW
 2020 OTE(IK)=SOTR(IK)/O2SUP*100.
C
C..... WRITE THE RESULTS
      WRITE(6,6120) (RK(IR),IR=1,3)
      WRITE(6,6220) ((KLAT(IP,IK),IK=1,2),IP=1,3)
      WRITE(6,6300) (CST(IP),IP=1,4),(TEMP(IT),PRES(IT),IT=1,4),
     1              PRES(5),WDEP,PRES(6),QAIR(JC),VW
      WRITE(6,6400) DE,QAIRS(JC)
      WRITE(6,6600) CS20,CBOOK
      WRITE(6,6520) (KLAS(IK),IK=1,2)
      WRITE(6,6800) UTRS,UTRE
      WRITE(6,6530) (KLAS(IK),IK=3,4)
      WRITE(6,6720) (SOTR(IK),IK=1,4),(OTE(IK),IK=1,2)
C.. PREPARE TO PLOT
        QAIRP(IC,NC(IC))=QAIRS(JC)
        CSSP(IC,NC(IC))=CS20
        DO 2030 IK=1,2
        KLAP(IC,NC(IC),IK)=KLAS(IK+2)
        SOTRP(IC,NC(IC),IK)=SOTR(IK+2)
 2030   OTEP(IC,NC(IC),IK)=OTE(IK)
        UTRP(IC,NC(IC),1)=UTRS
        UTRP(IC,NC(IC),2)=UTRE
C
 3000 CONTINUE
C
 4000   JCN=JC-1
C
 5100 FORMAT(1X,I3,5F9.0,3F7.0,F5.0,/,4X,2F9.0,5F7.0,/,4X,7F8.0,I5)
 5200 FORMAT(12A4)
 5300 FORMAT (10I3)
 6000 FORMAT (1H1,////,10X,'RUN # ',I3,' ... ',//)
 6110 FORMAT (10X,'UPTAKE RATE    = ',F7.2,'    MG/L/HR',//)
 6120 FORMAT (10X,'UPTAKE RATE PARAMETERS = ',F10.4,2F9.2,/)
 6200 FORMAT (10X,'KLAT 1          =',F9.4,10X,'KLAT 2          =',F9.4,//,
     1 10X,'KLAT MEAN     =',F9.4,10X,'% VARIATION    =',F6.2,//)
 6220 FORMAT (10X,'KLAT 1          =',F9.4,' * R + ',F9.4,/,
     1         10X,'KLAT 2          =',F9.4,' * R + ',F9.4,//,
     2         10X,'KLAT MEAN       =',F9.4,' * R + ',F9.4,//)
 6300 FORMAT (10X,'C*T 1           =',F7.2,12X,'C*T 2           =',F7.2,//,
     1 10X,'C*T MEAN      =',F7.2,12X,'% VARIATION    =',F7.2,///,
     2 10X,'TEMPERATURE    ',19X,     'PRESSURE',//,
     3 13X,'WATER      =',F7.2,' C',13X,'BAROMETRIC =',F7.2,' MMHG',//,
     4 13X,'COMP. AIR  =',F7.2,' F',13X,'          1 =',F7.2,' INHG',//,
     5 13X,'AMBIENT    =',F7.2,' C',13X,'          2 =',F7.2,' INH2O',//,
     6 13X,'INLET AIR  =',F7.2,' C',13X,'          3 =',F7.2,' INH2O',//,
```

```
     7 47X,'VAPOR  WT  =',F7.2,' MMHG',//,
     8 10X,'WATER DEPTH   =',F7.2,' FT',
     9                              12X,'VAPOR  IAT =',F7.2,' MMHG',//,
     1 10X,'QIAR          =',F7.2,' CFM',
     2                         8X,'WATER VOLUME  =',F7.2,' CU-FT')
 6400 FORMAT (////,
     1 30X,'*** RESULT OF EXPERIMENT ***',///,
     2 10X,'EFFECT. DEPTH =',F7.2,' FT',
     3 9X,'QAIR AT STP   =',F7.3,' SCFM',/)
 6500 FORMAT (10X,'KLA 20          =',F9.4,' 1/MIN',/)
 6510 FORMAT (10X,'KLA 20          =',F9.4,' 1/MIN AT',F8.4,
     1        ' MG/L/MIN UTR',/)
 6520 FORMAT (10X,'KLA 20          =',F9.4,' * R + ',F9.4,' 1/MIN',/)
 6530 FORMAT (15X,'KLA 20          =',2(2X,F9.4),'  1/MIN',/)
 6600 FORMAT (10X,'C* 20           =',F7.2,12X,
     1        'C* BOOK      =',F7.2,' MG/L',/)
 6700 FORMAT (10X,'SOTR            =',F9.3,' G/MIN',4X,
     1            'OTE             =',F9.3,' %',//,
     2        10X,'                =',F9.3,' IB/HR/1000 CU-FT')
 6720 FORMAT (15X,'SOTR            =',2(2X,F9.3),'  G/MIN',//,
     1        30X,                 2(2X,F9.3),'  IB/HR/1000 CU-FT',//,
     2        15X,'OTE             =',2(2X,F9.4),'  %')
 6800 FORMAT (//,15X,'UPTAKE RATE   =',2(2X,F9.3),'  MG/L/MIN',/,
     1           34X,20('_'),/)
C
C.. PLOT KLA VS. AIR FLOW RATE
      DO 7000 IP=1,3
      READ (5,5300) (ICP(I),I=1,5)
      XT(1)=0.
      XT(2)=3.
      YT(1)=0.
      YT(2)=.5
      READ (5,5200) (ITITLE(I),I=1,12)
      CALL CPLOT(3,1.,2,-3,XT,YT,1,1,L)
      DO 7200 I=1,5
        IF (ICP(I).EQ.0) GO TO 7200
        IC=ICP(I)
      NPL=NC(IC)
      IF (NPL.EQ.0) GO TO 7200
      DO 7210 J=1,NPL
      XT(J)=QAIRP(IC,J)
 7210 YT(J)=KLAP(IC,J,1)
      CALL CPLOT(3,1.,NPL,3,XT,YT,1,1,IC)
 7200 CONTINUE
 9999 STOP
      END


      FUNCTION CINTR(XARG)
C      THIS FUNCTION INTERPOLATES DO SATURATED VALUES IN BOOK
      REAL X(35),Y(35)
      DATA Y/15*0.,9.85,9.65,9.45,9.26,9.07,  8.90,8.72,8.56,8.40,8.24,
```

```
     1             8.09,7.95,7.81,7.67,7.54,  7.41,7.28,7.16,7.05,6.93/
      DO 1 J=1,35
 1    X(J)=FLOAT(J)
      J=IFIX(XARG)
      FACTOR=1.0
      JMIN=J-1
      IF (JMIN.GT.32) JMIN=32
      JMAX=JMIN+3
      DO 2 J=JMIN,JMAX
      IF (XARG.NE.X(J)) GO TO 2
      CINTR=Y(J)
      RETURN
 2    FACTOR=FACTOR*(XARG-X(J))
C.......INTERPOLATE DO VALUES BY LAGRANGE INTERPOLATION
      YEST=0.
      DO 4 I=JMIN,JMAX
      TERM=Y(I)*FACTOR/(XARG-I)
      DO 3 J=JMIN,JMAX
 3    IF (I.NE.J) TERM=TERM/(X(I)-X(J))
 4    YEST=YEST+TERM
      CINTR=YEST
      RETURN
      END
```

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