HELICON WAVES AT LOW MAGNETIC FIELDS

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ABSTRACT

In recent helicon wave experiments by our group and by T. Shoji of Nagoya University, it was found that a second, usually larger, peak in density occurred at very low magnetic fields. This peak cannot be caused by the lower hybrid resonance because at low fields the lower hybrid frequency is even farther from the operating frequency and the effects of ion motion are even more negligible. By retaining electron momentum in the equation of motion, we have found a second root of the dispersion relation which dominates at low fields. This root is related to the electrostatic electron cyclotron wave in a magnetic field; that is, to the lower Trivelpiece-Gould mode in cylindrical plasmas when $w_p >> w_c$. At magnetic fields of interest, the new mode is closely coupled to the usual helicon mode and becomes partly electromagnetic. Our treatment includes the gyratory motion of electron fluid elements in the magnetic field but neglects finite Larmor radius effects caused by thermal motions. The formalism also treats damping properly when it is not small. The simple helicon mode is recovered in the limit of small w/w_c, while the second mode disappears, its radial wavelength becoming infinitesimal. The new mode is of particular value to plasma sources because of the small B-fields needed to achieve high densities.

I. INTRODUCTION

The work of Boswell and co-workers (1982, 1984, 1985, 1987a, 1987b) and of Perry and Boswell (1989) has shown that excitation of helicon waves can lead to efficient generation of high-density plasmas. Chen (1985, 1988) has proposed that the efficient absorption of rf energy in these experiments is due to Landau damping and that plasma sources can be designed so that the trapping and acceleration of electrons in such waves produce primary electrons of the optimum energy. Recent experiments [Chen (1989), Chen and Decker (1989, 1990)] have given credence to this hypothesis. The dispersion relation for simple helicon waves of the type discussed by Woods (1962, 1964), Harding and Thonemann (1965a,b), and Lehane and Thonemann (1965) requires that the density $\underline{n}$ be proportional to the magnetic field $\underline{B}$ when all else is held constant. This basic relationship has been verified by Boswell and his co-workers, by Chen and his co-workers, and by T. Shoji of Nagoya University (1988).

In varying the ion mass, however, Shoji (1988) found that the linear increase of $\underline{n}$ with $\underline{B}$ occurred only with the heavier gases A and Xe. In Ne, a well-defined density peak occurred near the lower hybrid frequency (B = 590 G for f = 8.5 MHz). In H_2, D_2, and He, however, the peak occurred at a field (150-200 G) which was near the lower hybrid resonance but which did not vary with ion mass. Recently Chen and Decker (1989, 1990) have observed a significant peak in argon at very low fields (10-50 G). For low ion masses, one would expect that the simple helicon relation calculated with infinite ion mass would be affected by ion motions when the the lower hybrid frequency approaches the rf frequency. The low-field peak in argon, however, occurs when the lower hybrid frequency is even farther from the rf frequency than at the normal magnetic field for helicon waves and must, therefore, be caused by an effect other than finite ion mass.

When finite electron inertia is included in the theory of helicon waves, Klozenberg et al. (1965) found that there is a second root to the dispersion relation. We find that this root is essentially an electrostatic cyclotron wave propagating at an angle to the magnetic field (Trivelpiece-Gould mode). The new wave has a frequency below but near the electron cyclotron frequency and can be responsible for the low-field density peak that is observed. If so, a new type of plasma source can be designed which requires very little power in the magnetic field circuit.

II. THE FOURTH-ORDER WAVE EQUATION

We consider a cold plasma with stationary ions which fills uniformly a cylinder of radius $\underline{a}$ containing a uniform, coaxial magnetic field $\underline{B}_0 = B_0 z$. The electrons have no zero-order drift but in their wave motion have a phenomenological collision rate . The first-order equation of motion is then

$$m \frac{\partial \underline{v}}{\partial t} = -e(\underline{E} + \underline{v} \times \underline{B}_0) - m\nu \underline{v} . \tag{1}$$

The only other equations required are Maxwell's equations with displacement current neglected:

$$\underline{\nabla} \times \underline{E} = -\dot{\underline{B}} \qquad (\underline{\nabla} \cdot \underline{B} = 0) \tag{2}$$

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{j} \qquad (\underline{\nabla} \cdot \underline{j} = 0) . \tag{3}$$

Since the ions do not move, the plasma current is given by

$$\underline{j} = -n_0 e \underline{v} . \tag{4}$$

The resistivity η is related to ν by

$$\eta = \frac{m\nu}{n_0 e^2} . \tag{5}$$

Replacing $\underline{v}$ with $\underline{j}$ and ν with η, we can solve Eq.(1) for E, assuming perturbations of the form exp(-iωt), obtaining

$$\underline{E} = \frac{1}{en_0}(\underline{j} \times \underline{B}_0) + \eta\left(1 - \frac{i\omega}{\nu}\right)\underline{j} . \tag{6}$$

The first term in Eq.(6) describes the electron $\underline{E}$ x $\underline{B}$ drift and contributes only to $E_\perp$. The second term in Eq.(6), which gives all of $E_\parallel$ but only a small part of $E_\perp$, consists of two parts. The $\eta \underline{j}$ term is Ohm's law along $\underline{B}_0$ and cross-field mobility across $\underline{B}_0$. The iω term is actually independent of collisions and represents electron inertia along $\underline{B}_0$ and polarization drift across $\underline{B}_0$. This last drift, coupled with the $\underline{E}$ x $\underline{B}$ drift, describes the cyclotron motion of the electrons. Eqs.(2-3) and (6) are sufficient to determine the wave variables $\underline{E}$, $\underline{B}$, and $\underline{j}$. We follow the procedure of Klozenberg et al. (1965).

Substituting for $\underline{j}$ from Eq.(3), taking the curl of Eq.(6), and using this in Eq.(2), we obtain the following equation for the fluctuating magnetic field $\underline{B}$:

$$(\omega + i\nu)\,\underline{\nabla}\times\underline{\nabla}\times\underline{B} - \frac{e}{m}kB_o\,\underline{\nabla}\times\underline{B} + \frac{n_o e^2}{m}\,\omega\mu_o\underline{B} = 0. \tag{7}$$

Eq.(7) contains the assumption that $\underline{B}$ varies as exp[i(kx - wt)]. With the definitions

$$\omega_c \equiv \frac{eB_o}{m}, \qquad \omega_p^2 \equiv \frac{n_o e^2}{\epsilon_o m}, \tag{8}$$

Eq. (7) becomes

$$(\omega + i\nu)\,\underline{\nabla}\times\underline{\nabla}\times\underline{B} - k\omega_c\,\underline{\nabla}\times\underline{B} + (\omega\omega_p^2/c^2)\,\underline{B} = 0. \tag{9}$$

This can be factored into

$$(\beta_1 - \underline{\nabla}\times)(\beta_2 - \underline{\nabla}\times)\,\underline{B} = 0, \tag{10}$$

where β_1 and β_2 are the roots of the quadratic

$$(\omega + i\nu)\beta^2 - k\omega_c\beta + \omega\omega_p^2/c^2 = 0. \tag{11}$$

Let the solution of Eq. (10) be the sum of $\underline{B}_1$ and $\underline{B}_2$, which satisfy the equations

$$\underline{\nabla}\times\underline{B}_1 = \beta_1\underline{B}_1, \qquad \underline{\nabla}\times\underline{B}_2 = \beta_2\underline{B}_2. \tag{12}$$

Since the linear differential operators (β_j - ∇ x) commute, it is clear that $\underline{B} = \underline{B}_1 + \underline{B}_2$ satisfies Eq. (10):

$$(\beta_1 - \nabla\times)(\beta_2 - \nabla\times)\,\underline{B}_2 + (\beta_2 - \nabla\times)(\beta_1 - \nabla\times)\,\underline{B}_1 = 0.$$

$\underline{B} = \underline{B}_1 + \underline{B}_2$ is also the most general solution. From Eq.(2) we see that $\nabla\cdot(\underline{B}_1 + \underline{B}_2) = 0$, but $\nabla\cdot\underline{B}_1$ and $\nabla\cdot\underline{B}_2$ do not necessarily vanish separately. Taking the curl of Eqs.(12), we have

$$\underline{\nabla}(\underline{\nabla}\cdot\underline{B}_1) - \nabla^2\underline{B}_1 = \beta_1^2\underline{B}_1\,, \qquad \underline{\nabla}(\underline{\nabla}\cdot\underline{B}_2) - \nabla^2\underline{B}_2 = \beta_2^2\underline{B}_2\,.$$

Addition gives $\quad \nabla^2\underline{B}_1 + \beta_1^2\underline{B}_1 = -(\nabla^2\underline{B}_2 + \beta_2^2\underline{B}_2)$,

since $\underline{\nabla}[\underline{\nabla}\cdot(\underline{B}_1+\underline{B}_2)] = 0$. Since $\underline{B}_1$ and $\underline{B}_2$ must be different functions if $\beta_1 \neq \beta_2$, each side must vanish individually; and, hence, $\underline{\nabla}\cdot\underline{B}_1$ must also vanish for each wave. Thus, $\underline{B}_1$ and $\underline{B}_2$ satisfy

$$\nabla^2\underline{B}_1 + \beta_1^2\underline{B}_1 = 0\,, \qquad \nabla^2\underline{B}_2 + \beta_2^2\underline{B}_2 = 0. \tag{13}$$

The constants β_1 and β_2 can be written more succinctly by defining the complex constants

$$\alpha \equiv \frac{\omega}{k}\frac{n_0 e\mu_0}{B_0} = \frac{\omega}{k}\frac{\omega_p^2}{\omega_c c^2} \tag{14}$$

$$\gamma \equiv \frac{\omega + i\nu}{k\omega_c}\,. \tag{15}$$

The roots of Eq.(11) can then be written

$$\beta_{1,2} = \frac{1}{2\gamma}\left[1 \mp (1-4\alpha\gamma)^{1/2}\right]. \tag{16}$$

The usual helicon mode is β_1, with the minus sign, in the limit $\gamma \to 0$. Expanding the square root, we see that

$$\beta_1 \simeq \frac{1}{2\gamma}(2\alpha\gamma + 2\alpha^2\gamma^2) = \alpha(1+\alpha\gamma). \tag{17}$$

When $\nu >> \omega$, the inertia term can be neglected, and we have

$$\beta_1 = \alpha\left(1 + \frac{i\alpha\nu}{k\omega_c}\right) = \alpha(1+i\delta), \tag{18}$$

where

$$\delta = \frac{\alpha\nu}{k\omega_c} = \frac{\alpha}{k}\frac{\eta n_0 e^2}{m}\frac{m}{eB_0} = \frac{\eta\alpha^2}{\omega\mu_0}. \tag{19}$$

Eqns.(13a) and (18) are precisely the equations treated previously [Chen, 1985)].

The new root, β_2, takes the plus sign in Eq.(16), and its nature can be seen in the limit of small B_o, so that γ is large and α is small. If $4\alpha\gamma \ll 1$, Eq. (16) gives

$$\beta_2 \simeq \frac{1}{\gamma} = \frac{k\omega_c}{\omega + i\nu}. \tag{20}$$

For $\nu = 0$ and $\beta_2{}^2 = k_{tot}{}^2 = k_\perp{}^2 + k_\parallel{}^2$, the frequency is

$$\omega = (k_\parallel / k_{tot})\omega_c = \omega_c \cos\theta. \tag{21}$$

This is just the electrostatic electron cyclotron wave in a magnetic field, or the lower Trivelpiece-Gould mode in a cylinder when $\omega_p{}^2 \gg \omega_c{}^2$.

The waves B_1 and B_2, however, are not always so well separated. The two waves merge when $4\alpha\gamma = 1$. This condition, from Eqs.(14) and (15), can be written

$$\frac{\omega_p}{\omega_c} = \frac{1}{2}\frac{c}{v_\phi}, \qquad v_\phi = \frac{\omega}{k}. \tag{22}$$

Let us take $mv_\phi{}^2/2 = 50$ eV, the optimum energy for the ionizing electrons in argon. Eq.(22) then can be written

$$\frac{n_{14}}{B_3} = 1.23, \tag{23}$$

where, as usual, n_{14} is n_o in units of 10^{14} cm^{-3} and B_3 is B_o in kG. For $n_o = 2 \times 10^{12}$ cm^{-3}, we see that the critical field is 128 G. As B_o is lowered toward this value, the effect of electron inertia cannot be ignored in the helicon wave. At fields much lower than this, the electron cyclotron wave is dominant.

III. SOLUTION OF THE SECOND-ORDER WAVE EQUATIONS

Each of the two waves satisfies the equation

$$\nabla^2 \underline{B} + \beta^2 \underline{B} = 0 \tag{24}$$

and therefore will have the same solution but with a different value of β. In cylindrical coordinates, the components of Eq.(24)

become separable if $\underline{B}$ is expressed in terms of its right- and left-hand circular-polarization components given by

$$\sqrt{2}\, B_R = B_r - iB_\theta, \quad \sqrt{2}\, B_L = B_r + iB_\theta, \quad B_z = B_z. \tag{25}$$

The inverse transformation is

$$\sqrt{2}\, B_r = B_R + B_L, \quad \sqrt{2}\, B_\theta = i(B_R - B_L), \quad B_z = B_z. \tag{26}$$

For $B \propto \exp[i(m\theta + kz - \omega t)]$, Eq. (24) then becomes

$$\left\{\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \left[1 - \frac{(m-1)^2}{\rho^2}\right]\right\} B_R = 0 \tag{27a}$$

$$\left\{\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \left[1 - \frac{(m+1)^2}{\rho^2}\right]\right\} B_L = 0 \tag{27b}$$

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \left(1 - \frac{m^2}{\rho^2}\right)\right] B_z = 0, \tag{27c}$$

where $\rho \equiv Tr, \qquad T^2 \equiv \beta^2 - k^2.$ (28)

The solutions of these Bessel equations that are finite at the origin are:

$$B_R = C_1' J_{m-1}(Tr) \tag{29a}$$

$$B_L = C_2' J_{m+1}(Tr) \tag{29b}$$

$$B_z = C_3 J_m(Tr). \tag{29c}$$

Since Eqs.(2) and (3) state that $\nabla \times \underline{E} = i\omega(\underline{B}_1 + \underline{B}_2)$ and $\mu_0 \underline{j} = \nabla \times (\underline{B}_1 + \underline{B}_2)$, we may define $\underline{E}_1$, $\underline{E}_2$, $\underline{j}_1$, and $\underline{j}_2$ such that $\underline{E} = \underline{E}_1 + \underline{E}_2$ and $\underline{j} = \underline{j}_1 + \underline{j}_2$; then Eqs. (2) and (3) are satisfied for each wave individually. From Eqs.(3) and (12), we see that

$$\underline{j} = (\beta/\mu_0)\,\underline{B}. \tag{30}$$

The coefficients C_1, C_2, and C_3 may now be found (for each wave separately) from $\nabla \cdot \underline{B} = 0$ and by comparing the results of evaluating $\underline{E}_j$ in terms of $\underline{B}_j$ (j = 1,2) in two ways: from Eq.(6) and (30), and from Eq.(2). Dropping the subscript j, we have from Eq.(26) the B components

$$B_r = C_1 J_{m-1} + C_2 J_{m+1} \tag{31a}$$

$$B_\theta = i\,(C_1 J_{m-1} - C_2 J_{m+1}) \tag{31b}$$

$$B_z = C_3 J_m \tag{31c}$$

where $C_1 = C_1' / \sqrt{2}$ and $C_2 = C_2' / \sqrt{2}$,

and the argument Tr of the Bessel functions is understood. The condition $\nabla \cdot \underline{B} = 0$ is

$$B_r' + \frac{B_r}{r} + \frac{im}{r} B_\theta + ik B_z = 0, \tag{32}$$

where the prime (') denotes $\partial / \partial r$. With Eq.(31), this becomes

$$C_1 J_{m-1}' + \frac{1}{r} C_1 J_{m-1} + C_2 J_{m+1}' + \frac{1}{r} C_2 J_{m+1} - \frac{m}{r}(C_1 J_{m-1} - C_2 J_{m+1}) + ik C_3 J_m = 0. \tag{33}$$

Wsing the recursion relations

$$J_n'(Tr) = T\left(J_{n-1} - \frac{n}{Tr} J_n\right) = -T\left(J_{n+1} - \frac{n}{Tr} J_n\right), \tag{34}$$

we find

$$C_1\left[-TJ_m + \frac{m-1}{r} J_{m-1} + \frac{1-m}{r} J_{m-1}\right] + C_2\left[TJ_m - \frac{m+1}{r} J_{m+1} + \frac{m+1}{r} J_{m+1}\right] + ikC_3 J_m = 0, \tag{35}$$

$$C_3 = -i\,\frac{T}{k}\,(C_1 - C_2)\,. \tag{36}$$

Next we consider Eq.(6). Substituting Eq.(30) into Eq.(6), we obtain

$$\underline{E} = \frac{\omega}{k}\frac{\beta}{\alpha}\,\underline{B}\times\hat{z} + \eta\left(1-\frac{i\omega}{\nu}\right)\frac{\beta}{\mu_0}\,\underline{B}\,, \tag{37}$$

where

$$\eta\left(1-\frac{i\omega}{\nu}\right)\frac{\beta}{\mu_0} = -i\,\frac{\omega+i\nu}{\nu}\,\frac{m\nu\beta}{n_0 e^2\mu_0} = -i\frac{\omega}{k}\frac{\beta}{\alpha}\,\frac{\omega+i\nu}{\omega_c}\,. \tag{38}$$

It will be convenient to define

$$a_1 \equiv \frac{\omega}{k}\frac{\beta}{\alpha}\,, \quad a_2 \equiv \frac{\omega}{k}\frac{\beta}{\alpha}\,\frac{\omega+i\nu}{\omega_c} = a_1\,\frac{\omega+i\nu}{\omega_c} \tag{39}$$

and

$$a_+ \equiv a_1 + a_2\,, \qquad a_- \equiv a_1 - a_2\,. \tag{40}$$

Eqs.(37) and (31) then yield

$$E_r = i\left(a_- C_1 J_{m-1} - a_+ C_2 J_{m+1}\right) \tag{41a}$$

$$E_\theta = -\left(a_- C_1 J_{m-1} + a_+ C_2 J_{m+1}\right) \tag{41b}$$

$$E_z = -i a_2 C_3 J_m\,. \tag{41c}$$

The rotating components are given succinctly by

$$E_R = i a_- B_R\,, \qquad E_L = -i a_+ B_L\,, \qquad E_z = -i a_2 B_z\,. \tag{42}$$

Finally, we use Eq.(2), whose components are

$$B_r = \frac{1}{\omega}\left(\frac{m}{r}E_z - kE_\theta\right) \tag{43a}$$

$$B_\theta = \frac{1}{\omega}\left(kE_r + iE_z'\right) \tag{43b}$$

$$B_z = -\frac{i}{r\omega}(rE_\theta)' - \frac{m}{r\omega}E_r . \tag{43c}$$

Inserting the solutions Eqs.(31) and (41) into Eq.(43), we obtain

$$\left(1-\frac{k}{\omega}a_-\right)C_1 J_{m-1} + \left(1-\frac{k}{\omega}a_+\right)C_2 J_{m+1} + i\,\frac{m a_z}{r\omega} C_3 J_m = 0 \tag{44a}$$

$$\left(1-\frac{k}{\omega}a_-\right)C_1 J_{m-1} - \left(1-\frac{k}{\omega}a_+\right)C_2 J_{m+1} + i\,\frac{a_z}{\omega} C_3 J_m' = 0 \tag{44b}$$

$$T\left(a_- C_1 - a_+ C_2\right)J_m - i\omega C_3 J_m = 0, \tag{44c}$$

where we have used the recursion relations [Eq.(34)]. The sum and difference of Eqs.(44a,b) then yield the following relations:

$$2\left(1-\frac{k}{\omega}a_-\right)C_1 J_{m-1} + \frac{i}{\omega}a_z C_3\left(J_m' + \frac{m}{r}J_m\right) = 0 \tag{45}$$

$$2\left(1-\frac{k}{\omega}a_+\right)C_2 J_{m+1} - \frac{i}{\omega}a_z C_3\left(J_m' - \frac{m}{r}J_m\right) = 0 \tag{46}$$

The Bessel functions cancel out according to Eq.(34), giving

$$C_3 = \frac{2i\omega}{a_z T}\left(1-\frac{k}{\omega}a_-\right)C_1 = \frac{2i\omega}{a_z T}\left(1-\frac{k}{\omega}a_+\right)C_2 . \tag{47}$$

Thus

$$C_2 = C_1\left(1-\frac{k}{\omega}a_-\right)/\left(1-\frac{k}{\omega}a_+\right). \tag{48}$$

Eqs. (36) and (44) also give

$$C_3 = -i\frac{T}{k}\left(C_1 - C_2\right) = -i\frac{T}{\omega}\left(a_- C_1 - a_+ C_2\right). \tag{49}$$

These relations are all consistent with one another. Indeed, Eqs. (2) and (6) are six scalar equations for the two ratios C_2/C_1 and C_3/C_1, but the superfluous equations simply reduce to the known relations Eqs.(11) and (28). Eqs.(48) and (41), then, give all the wave components in terms of a single amplitude C_1. In terms of more familiar variables, these relations are

$$C_2 = C_1 \frac{\alpha - \beta[1-(\omega+i\nu)/\omega_c]}{\alpha - \beta[1+(\omega+i\nu)/\omega_c]} \tag{50}$$

$$C_3 = C_1 \frac{2iT}{k} \frac{\beta(\omega+i\nu)/\omega_c}{\alpha-\beta[1+(\omega+i\nu)/\omega_c]}, \tag{51}$$

where α and β are given by Eqs.(14) and (16). An alternate expression for C_3 is

$$C_3 = C_1 \frac{2ik}{T}\left[1-\left(1-\frac{\alpha}{\beta}\right)\frac{\omega_c}{\omega+i\nu}\right]. \tag{52}$$

Before leaving this solution, we wish to estimate the extent to which each wave is electrostatic. The space charge is given by

$$\underline{\nabla}\cdot\underline{E} = \frac{1}{r}(rE_r)' + \frac{im}{r}E_\theta + ikE_z. \tag{53}$$

From Eqs.(34) and (41) we obtain

$$\underline{\nabla}\cdot\underline{E} = i\,(Ta_-C_1 + ka_2C_3 - Ta_+C_2)\,J_m(Tr). \tag{54}$$

This is in general quite a complicated expression, but if a_2 is small, we have $a_+ \approx a_-$ and thus $C_2 \approx C_1$. We then have

$$\begin{aligned}\underline{\nabla}\cdot\underline{E} &\simeq a_2(2TC_1 + ikC_3)J_m \\ &= 2i\frac{\beta}{\alpha}\frac{\omega T}{k}C_1J_m\left[\frac{k^2}{T^2}\left(\frac{\alpha}{\beta}-1\right)+\left(1-\frac{k^2}{T^2}\right)\frac{\omega+i\nu}{\omega_c}\right].\end{aligned} \tag{55}$$

Since $\beta \approx \alpha$ for mode 1 while $\beta > \alpha$ for mode 2, it is seen that mode 2 , which is basically an electrostatic cyclotron wave, has a much larger electrostatic component than mode 1, which is basically an electromagnetic whistler wave. Mode 1 helicon waves, nonetheless, have a sufficiently large electrostatic E-field to make their wave patterns almost orthogonal to those of electromagnetic waves in a vacuum waveguide.

IV. CHARACTERISTICS OF THE TWO WAVES

A) The Undamped Modes

The total wavenumber β is given by

$$\beta = \frac{1}{2\gamma}\left[1 \mp (1 - 4\alpha\gamma)^{1/2}\right], \tag{56}$$

where, for $\nu = 0$,

$$\gamma = \frac{1}{k}\frac{\omega}{\omega_c}, \qquad \alpha = \frac{\omega}{k}\frac{\omega_p^2}{\omega_c c^2}. \tag{57}$$

Note that α, β, and γ are functions only of n_o, B_o, and $v_o = w/k$, and γ is real in this section.

1. Condition for merged roots.

The two β's are identical if

$$4\alpha\gamma = 1 \qquad \text{or} \qquad \frac{\omega_p}{\omega_c}\frac{v_\phi}{c} = \frac{1}{2}. \tag{58}$$

For instance, if $n_o = 10^{13}\ cm^{-3}$, and $v_o = 4.19 \times 10^8$ cm/sec, corresponding to $E_o \equiv mv_o^2/2 = 50$ eV, then $w_c = 5 \times 10^9\ sec^{-1}$, or $B_o = 284$ G. The common value of β is

$$\beta = \frac{1}{2\gamma} = \frac{k}{2}\frac{\omega_c}{\omega} = \frac{1}{2}\frac{\omega_c}{v_o}. \tag{59}$$

Solving for v_o and inserting into Eq.(58), we obtain

$$\frac{\omega_p}{\omega_c}\frac{\omega_c}{c}\frac{1}{2T} = \frac{1}{2}, \qquad \frac{a}{p_{11}} = \frac{c}{\omega_p}. \tag{60)}$$

Here, we have taken $\beta \approx T$ and $T = p_{11}/a$, where p_{11} is the first zero of $J_1(Tr)$ and a is the tube radius. Taking $p_{11} = 3.83$, we obtain a critical tube radius

$$a = 0.20 / \sqrt{n_{14}} \quad \text{cm}, \tag{61}$$

which is independent of v_0 and ω_c. Since β is complex for $4\alpha\gamma > 1$, both waves are evanescent unless $4\alpha\gamma < 1$; but the condition is not as simple as Eq.(61) for the coalescence of roots.

2. Condition for separated roots.

The two roots are well separated if

$$\frac{\omega_p}{\omega_c}\frac{v_0}{c} \ll \frac{1}{2} \quad \text{or} \quad B_0 \gg 89.7\sqrt{n_{12}} \quad (\text{for } E_0 = 50 \text{ eV}) \tag{62}$$

In that case, we can give approximate dispersion relations for each.

a) Helicon wave. For small $4\alpha\gamma$, we may expand the square root in Eq.(56) to second order to obtain

$$\beta_1 \simeq \alpha(1+\alpha\gamma) = \frac{\omega}{k}\frac{\omega_p^2}{\omega_c c^2}\left[1 + \left(\frac{\omega}{ck}\frac{\omega_p}{\omega_c}\right)^2\right], \tag{63}$$

where the second term is necessarily small.

b) Cyclotron wave. Expanding Eq.(56) with the plus sign, we obtain

$$\beta_2 \simeq \frac{1}{\gamma} - \alpha = \frac{k\omega_c}{\omega}\left[1 - \left(\frac{\omega}{ck}\frac{\omega_p}{\omega_c}\right)^2\right]. \tag{64}$$

B) Boundary Conditions

1. Insulating boundary

For a non-conducting tube, the boundary condition at $r = a$ is

$$j_r = \frac{\beta}{\mu_0}B_r = 0. \tag{65}$$

a) Coupled modes. The two modes may have non-vanishing j_r's such that they cancel each other at the boundary. We then have

$$\beta_1 B_{1r} + \beta_2 B_{2r} = 0, \tag{66}$$

where

$$B_r = C_1 J_{m-1}(Ta) + C_2 J_{m+1}(Ta) \tag{67}$$

for each mode, and the β's are given by Eq.(56). It will be convenient to define

$$\begin{aligned} A_1 &= (C_1)_1\,, \quad R_1 = (C_2/C_1)_1 \qquad \text{for wave 1} \\ A_2 &= (C_1)_2\,, \quad R_2 = (C_2/C_1)_2 \qquad \text{for wave 2}\,. \end{aligned} \tag{68}$$

Then

$$\beta_1 A_1 (J_{m-1} + R_1 J_{m+1}) + \beta_2 A_2 (J_{m-1} + R_2 J_{m+1}) = 0, \tag{69}$$

where

$$T_j^2 = \beta_j^2 - k^2, \qquad R_j = \frac{\alpha - \beta_j(1-k\gamma)}{\alpha - \beta_j(1+k\gamma)}\,. \tag{70}$$

Both α and γ are fixed by n_0, B_0, w, and k (which had to be chosen at the outset) and do not depend on the wave in question. Then the β's are fixed by Eq.(56) and, thus, so are the T's and R's, which are different for the two modes. Eq.(69) then gives the ratio of the amplitudes, A_2/A_1.

b) <u>The merged mode</u>. Here $\beta_1 = \beta_2$, and thus $T_1 = T_2$, $A_1 = A_2$, and $R_1 = R_2$. Eq.(69) then gives

$$J_{m-1} + R J_{m+1} = 0 \qquad \text{(at } r = a\text{)}. \tag{71}$$

Since $4\alpha\gamma = 1$ here, we may use this in Eq.(70) to obtain

$$R = \frac{1-2k\gamma}{1+2k\gamma} = \frac{\omega_c - 2\omega}{\omega_c + 2\omega} < 1 \qquad (\text{for } \nu = 0). \tag{72}$$

We see that this mode is right-hand elliptically polarized.

polarized at the center, as discussed above.

The boundary condition for either wave is given by Eq.(71) with the corresponding value of R. By use of the recursion relations Eq.(34), the boundary condition can also be written

$$(1-R)J_m' + (1+R)\frac{m}{a}J_m = 0 \qquad \text{at } r = a. \tag{78}$$

From Eq.(75), we find

$$\frac{1-R}{1+R} = -\frac{k\beta\gamma}{\alpha-\beta}, \tag{79}$$

so that the boundary condition Eq.(78) can be written

$$\frac{m}{a}J_m = \frac{k\beta\gamma}{\alpha-\beta}J_m'. \tag{80}$$

In the limit of zero electron inertia, both γ and $\alpha - \beta$ approach zero. To recover our previous result for the helicon wave, we can substitute for β from Eq.(63) to obtain

$$\frac{m}{a}J_m = -\frac{k\beta\gamma}{\alpha^2\gamma}J_m' \simeq -\frac{k}{\alpha}J_m', \tag{81}$$

$$\frac{m}{a}\alpha J_m + kJ_m' = 0,$$

which is precisely the boundary condition we obtained earlier for the $w/w_c = 0$ case.

2. Conducting boundary

Now we require $E_z = 0$ and $E_\theta = 0$. From Eq.(41), we have

$$\begin{aligned} E_\theta &= -(a_-C_1J_{m-1} + a_+C_2J_{m+1}) \\ E_z &= -ia_2C_3J_m \end{aligned} \tag{82}$$

When $a_2 = a_1 \cdot (w + i\nu)/w_c$ is not zero, both boundary conditions cannot be satisfied. Since E_θ can be expressed as a sum of J_m and J_m' terms, both J_m and J_m' must vanish at the boundary, and that is not possible. The waves can nonetheless exist in a conducting cylinder because the plasma will not extend all the way to the wall. A thin vacuum layer near the wall will allow $E_{t,n}$ to be finite, and tangential surface currents will flow there. However, radial currents cannot flow through the vacuum, so the boundary acts like an insulating boundary. Unless the vacuum layer is thick, one does not have to match to the vacuum solution; the insulating boundary condition will be followed approximately.

In any case, we can show that the condition $E_\theta = 0$ does not differ much from the condition $j_r = 0$. It does not differ at all if $a_2 = 0$, the ideal helicon case. Defining

$$\epsilon \equiv k\gamma = \frac{\omega + i\nu}{\omega_c}, \tag{83}$$

we can write Eq.(82a) as

$$(1-\epsilon)C_1 J_{m-1} + (1+\epsilon)C_2 J_{m+1} = 0. \tag{84}$$

Converting to J_m and J_m' by Eq.(34) and using $R = C_1/C_2$, we obtain

$$\left(\frac{1-R}{1+R} - \epsilon\right) J_m' + \frac{m}{a}\left(1 - \epsilon\,\frac{1-R}{1+R}\right) J_m = 0. \tag{85}$$

Eq.(79) gives

$$\frac{1-R}{1+R} = -\frac{\epsilon\beta}{\alpha-\beta}, \tag{86}$$

so that Eq.(85) can be written

$$\epsilon\alpha J_m' = \frac{m}{a}(\alpha - \beta + \epsilon^2\beta) J_m. \tag{87}$$

In terms of γ, this becomes

$$\frac{m}{a} J_m = \frac{k\alpha\gamma}{\alpha - \beta + k^2\gamma^2\beta} J_m' \tag{88}$$

which differs from the insulating boundary condition Eq.(80) by only the last term in the denominator.

C) Excitation and Damping

1. Excitation

In the laboratory, the helicon and cyclotron branches will probably be excited as separate modes, satisfying the boundary condition separately. For simplicity, we discuss the collisionless case $\nu = 0$. Then

$$\beta_1 \simeq \alpha(1+\alpha\gamma) \simeq \alpha$$
$$\beta_2 \simeq \frac{1}{\gamma} + \alpha \simeq \frac{1}{\gamma}, \tag{89}$$

where

$$\beta^2 = T^2 + k^2. \tag{90}$$

If the lowest radial mode is excited for each wave, so that $T \approx p_{11}/a$, then k_1 will be different from k_2. Although a common k was chosen initially, we can take different k's when the waves are uncoupled. For $\nu = 0$, Eq.(11) becomes

$$\omega T^2 - k\omega_c T + \omega\omega_p^2/c^2 = 0 \tag{91}$$

where we have taken $\beta \approx T$. Solving for w/k, we obtain

$$\frac{\omega}{k} = v_0 = \frac{\omega_c T}{T^2 + \omega_p^2/c^2}. \tag{92}$$

For the helicon mode, using β_1 from Eq.(89) and assuming $\beta_1 \approx T$, we find

$$\frac{\omega}{k} = v_0 = \frac{\omega_c c^2}{\omega_p^2} T = \frac{\omega_c c^2}{p^2}\frac{p_{11}}{a}. \tag{93}$$

This is the same as Eq.(92), with $\omega_p^2/c^2 \gg T^2$. For the cyclotron mode, using β_2 from Eq.(89), we find

$$\frac{\omega}{k} = v_0 = \frac{\omega_c}{T} = \omega_c \frac{a}{p_{11}}, \tag{94}$$

which is the same as Eq.(92) with $\omega_p^2/c^2 \ll T^2$. Since both waves have v_0 proportional to B_0, it would seem that sweeping the magnetic field would not make the dominant mode jump from one to the other.

However, the observations can be explained as follows. Starting at large B_0, a density is reached which satisfies Eq.(93) for a value of v_0 that gives good production of primaries. The helicon mode is dominant. As the field is swept downwards, the density n_0 falls linearly with B_0. Eventually, the field becomes so low that Eq.(94) is satisfied for the optimum v_0; then the cyclotron wave is excited and the density rises again. At these low fields and high densities, w_c and w_p do not satisfy Eq.(93) any more, and the helicon wave is not excited.

2. Damping

a) Merged mode. For this mode we have $4\alpha\gamma = 1$, where α and γ are given by Eqs.(14) and (15). Solving for k^2, we obtain

$$k^2 = \frac{4\omega^2\omega_p^2}{\omega_c^2 c^2}\left(1+\frac{i\nu}{\omega}\right). \tag{95}$$

Another equation for k^2 arises from the condition

$$\beta^2 = T^2 + k^2 = \left(\frac{1}{2\gamma}\right)^2 = (2\alpha)^2 = \left(\frac{2\omega_p^2\omega}{k\omega_c c^2}\right)^2. \tag{96}$$

This leads to

$$k^4 + T^2k^2 - 4Q^2 = 0 \qquad \text{where} \qquad Q \equiv \frac{\omega\omega_p^2}{\omega_c c^2}. \tag{97}$$

Since Eq.(95) leads to complex k^2 whereas Eq.(97) gives real k^2, there can be no merged mode when there is dissipation.

b) Helicon mode. Here we have

$$\beta_1^2 = T^2 + k^2 \simeq [\alpha(1+\alpha\gamma)]^2 \tag{98}$$

where $\gamma = \epsilon/k, \quad \alpha = Q/k,$

with ϵ and Q defined by Eqs.(83) and (97), respectively. This leads to an 8th order equation for complex k:

$$k^8 + T^2k^6 - Q^2(k^4 + 2\epsilon Q k^2 + \epsilon^2 Q^2) = 0 \tag{99}$$

To get an approximate answer, we let $k = k_r + ik_i$ and ignore k_i in small terms. Eq.(98) then becomes

$$\beta_1 = \frac{Q}{k}\left(1+\frac{\epsilon Q}{k_r^2}\right) = (T^2+k_r^2)^{1/2} \equiv \beta_r . \tag{100}$$

This yields

$$k_r = \frac{Q}{\beta_r}\left(1+\frac{Q}{k_r^2}\frac{\omega}{\omega_c}\right) \tag{101}$$

$$k_i = \frac{Q^2}{\beta_r k_r^2}\,\frac{\nu}{\omega_c} . \tag{102}$$

The ratio is

$$\frac{k_i}{k_r} = \frac{\nu}{\omega}\,\frac{\omega^2}{\omega_c^2}\,\frac{\omega_p^2}{c^2 k_r^2} , \tag{103}$$

which reduces to our previous result $(\nu/\omega)(c^2T^2/\omega_p 2)$ when T is approximated by the first term of β_1 in Eq.(98).

c) Cyclotron mode. From Eq.(89b), we now have

$$\beta_2^2 = T^2+k^2 = \left(\frac{k}{\epsilon}-\frac{Q}{k}\right)^2 . \tag{104}$$

This is a 4th order equation for k. As an approximation, we define β_r as in Eq.(100), whereupon Eq.(104) becomes

$$k^2 - \beta_r \epsilon k - \epsilon Q = 0. \tag{105}$$

Again writing $k = k_r + ik_i$, and also $\epsilon = \epsilon_r + i\epsilon_i$, we can solve the real and imaginary parts of Eq.(105) to obtain

$$k_r = \tfrac{1}{2}\beta_r\epsilon_r\left[1+\left(1+\frac{4Q\epsilon_r}{\beta_r^2\epsilon_r^2}\right)^{1/2}\right] \simeq \beta_r\frac{\omega}{\omega_c}+\frac{Q}{\beta_r} , \tag{106}$$

$$k_i \simeq \frac{\nu}{\omega_c}\,\frac{\beta_r + Q/k_r}{2-\beta_r\omega/k_r\omega_c} , \tag{107}$$

where $\beta_r \approx T$. The ratio is

$$\frac{k_i}{k_r} \simeq \frac{\nu}{\omega_c} \frac{\beta_r}{2k_r - \beta_r \omega/\omega_c} \simeq \frac{\nu}{\omega_c} \frac{\beta_r}{k_r + Q/\beta_r} \simeq \frac{\nu}{\omega_c} \frac{T}{k_r} . \qquad (108)$$

IV. CONCLUSION

We have shown that the density peak at low magnetic fields in helicon-wave discharges can be explained by an electron cyclotron wave.

The following tasks remain to be done:

1. Calculate the energy absorption profile.

2. Find expressions for the electrostatic and electromagnetic parts of the E-field.

3. Incorporate the effect of an equilibrium E-field. In particular, the enhancement of the ambipolar E-field by a central electrode can cause the fast electrons to be confined in axis-encircling orbits with radii smaller than their Larmor radii.

4. Calculate the effect of a zero-order radial density gradient.

5. Calculate the distribution function of the fast electrons and consider their effect on the dispersion relation.

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