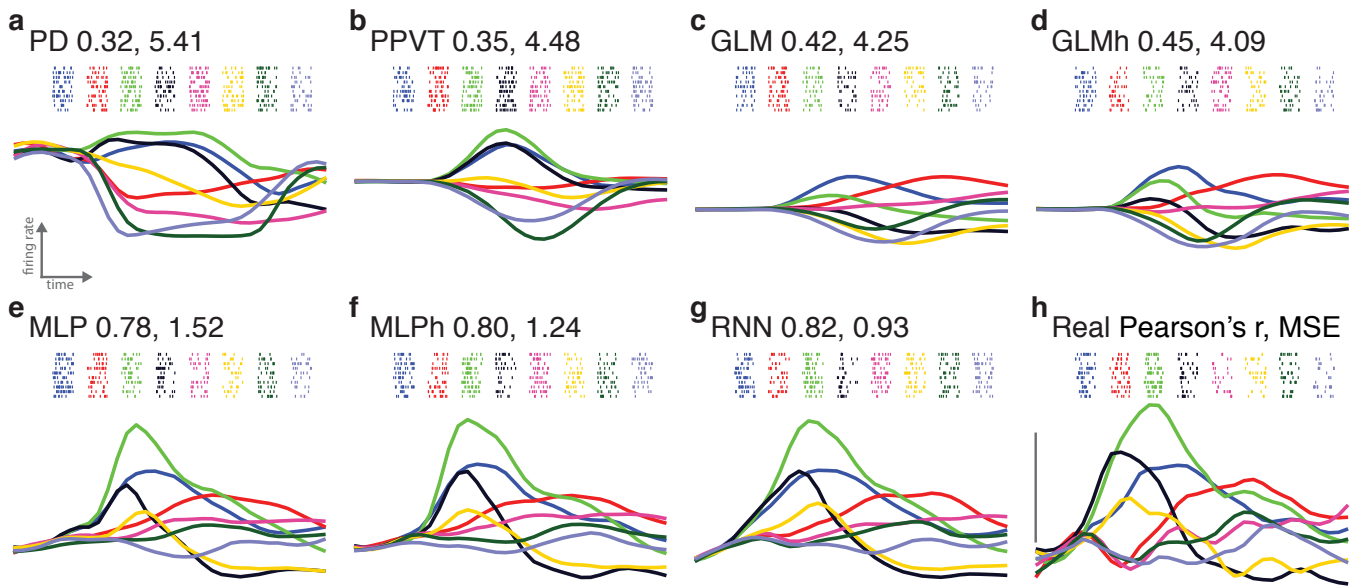
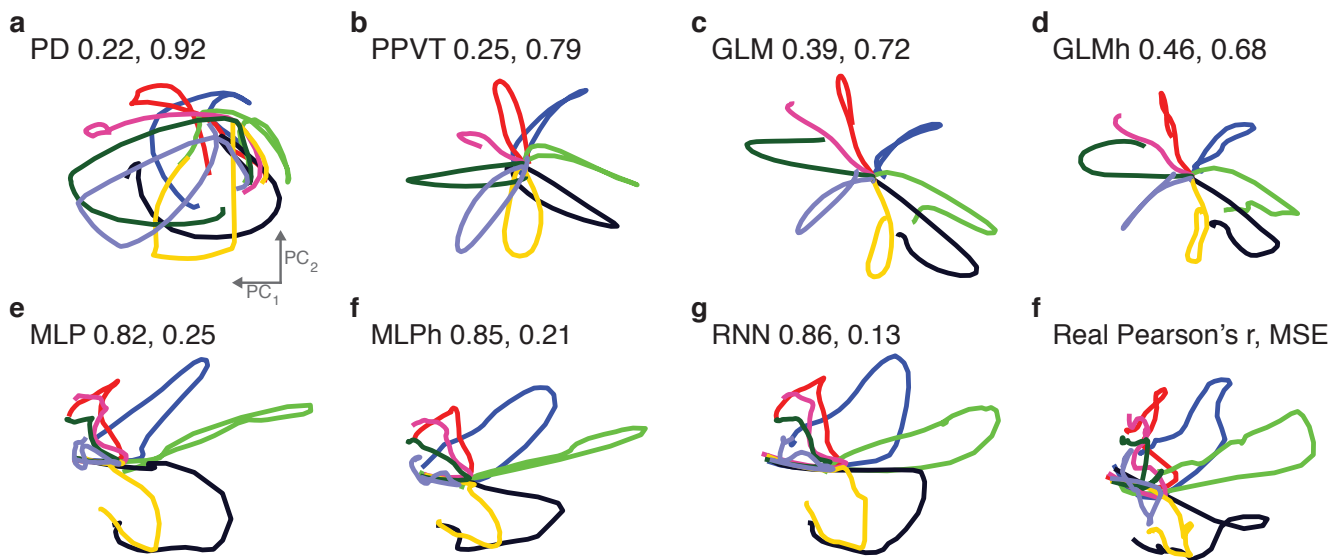


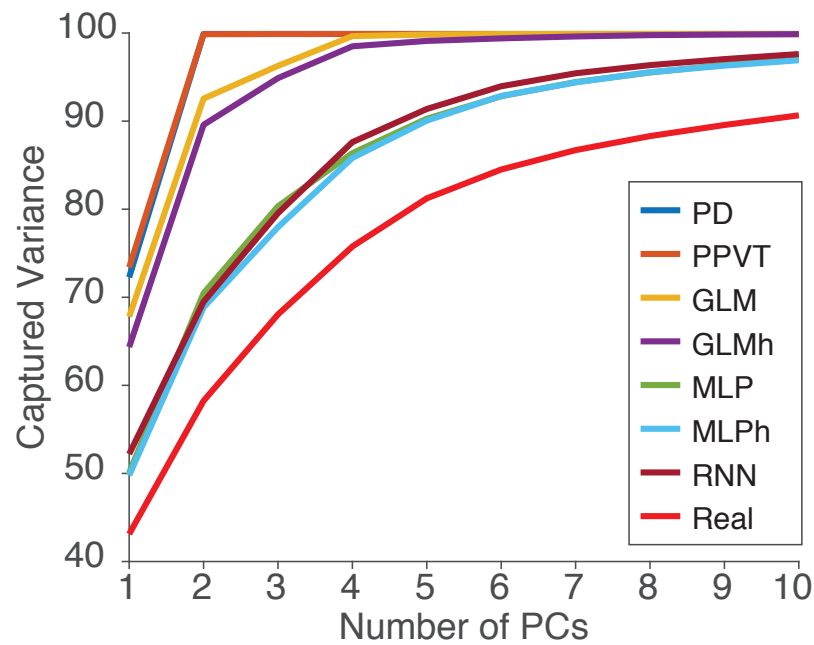
1 Supplementary Figures



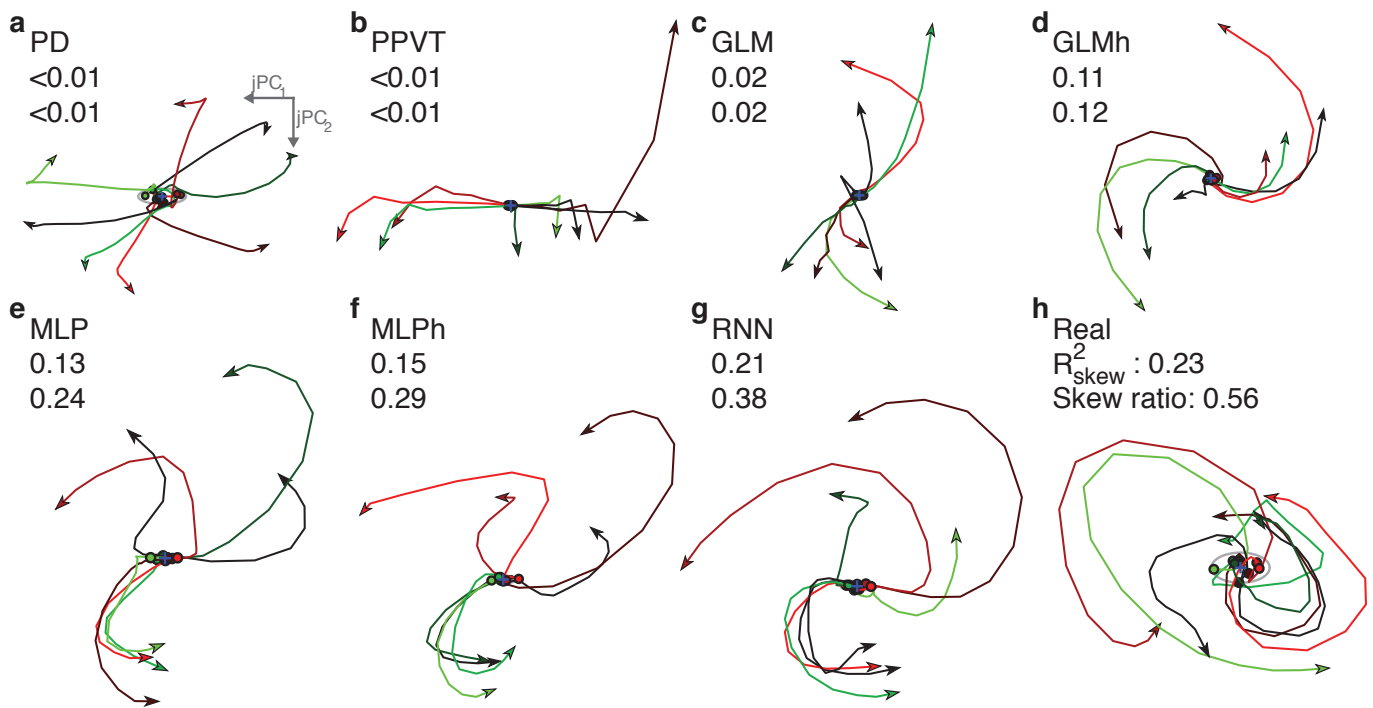
Supplementary Figure 1: PSTHs and spike rasters of recorded and synthetic neural activity. The PSTHs are of: **a** PD, **b** PPVT, **c** GLM, **d** GLMh, **e** MLP, **f** MLPh, **g** RNN, **h** a randomly chosen electrode from empirical recordings. **a-g** show the corresponding neural encoder outputs in reproducing electrode activity in **h**. Each color shows the average firing rate to one of eight center-out reach condition over time, with eight different colors corresponding to eight reach conditions. In **h**, the vertical bars denote 100 spikes/s, and the horizontal bar denotes 100 ms. The average PCC across all neurons and average MSE of PSTH (spikes²/s) for each encoding model are shown in each panel. We also show spike rasters in each panel. The PCC for single-trial binned spike counts for PD, PPVT, GLM, GLMh, MLP, MLPh and RNN were 0.02, 0.01, 0.03, 0.03, 0.07, 0.08, and 0.08, respectively. Their respective MSEs were 62.26, 58.62, 49.67, 49.39, 49.95, 49.51, and 50.78 spikes²/s.



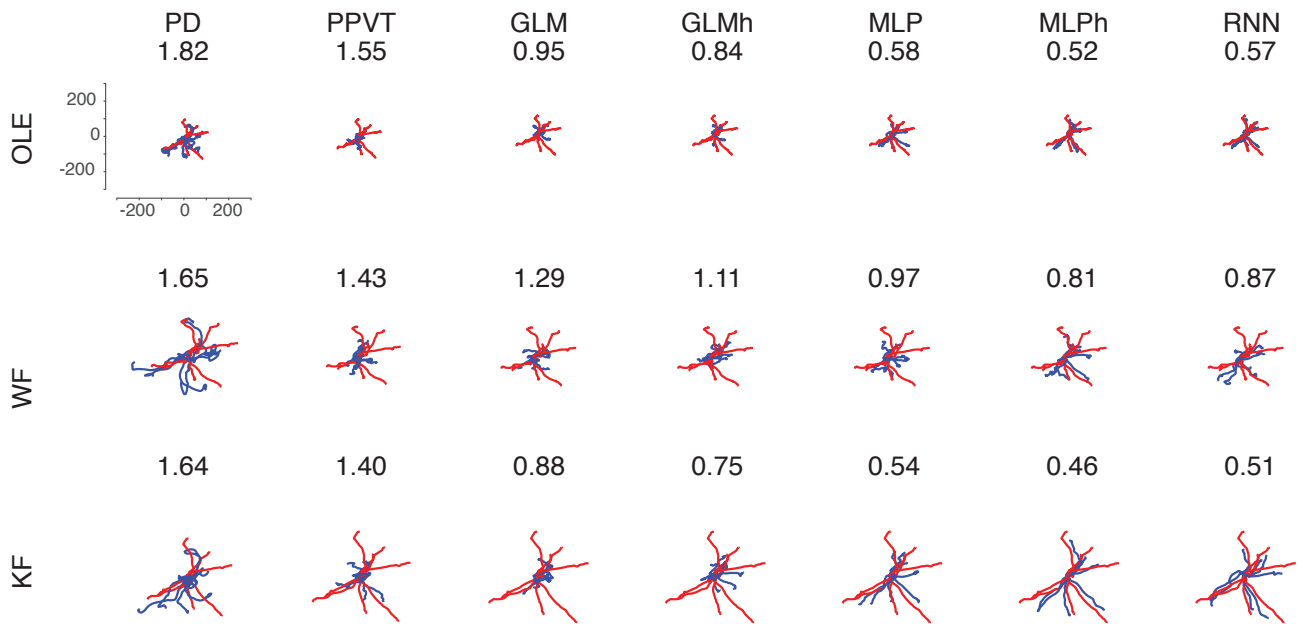
Supplementary Figure 2: Projections of PSTHs on PCs found from real neural data. The projections are of **a** PD, **b** PPVT, **c** GLM, **d** GLMh, **e** MLP, **f** MLPh, **g** RNN, **h** empirically recorded neural population activity. Each color shows the neural trajectory of one of eight center-out reach conditions over time. We calculate the MSE and PCC of neural trajectories in the top 10 PCs.



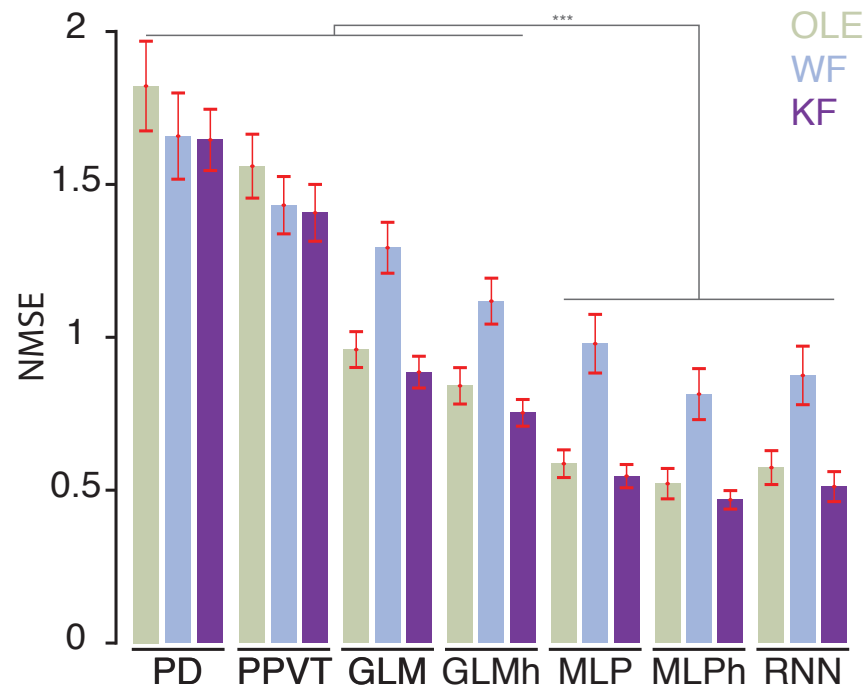
Supplementary Figure 3: Dimensionality of recorded and synthetic PSTHs. Only 2 PCs were needed to capture almost 100% variance of PD and PPVT activity. To capture over 90% of recorded neural variance, we required 2 PCs for GLM, 3 PCs for GLMh, and 5 PCs for DL models. The real data required 10 PCs.



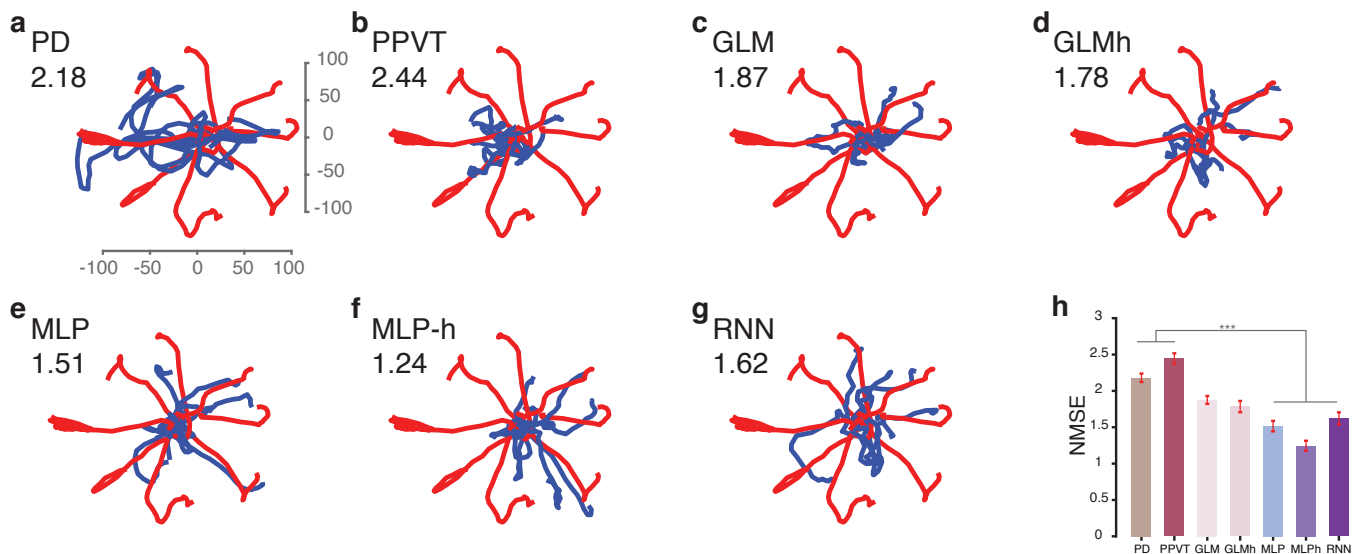
Supplementary Figure 4: jPCA projections of the population responses. Each plot shows the jPCA projection of eight center-out reach conditions (different colors). The jPCA projections are of: **a** PD, **b** PPVT, **c** GLM, **d** GLMh, **e** MLP, **f** MLPh, **g** RNN, **h** empirically recorded neural population activity. We show the R^2_{skew} in the jPCs as well as R^2_{skew}/R^2_{best} (Skew ratio) for each encoding model (see Methods).



Supplementary Figure 5: Decoded movements from real and synthetic neural activity with three standard decoders (OLE, WF, and KF). For each direction, one randomly chosen trial is shown. In each panel, red lines represent decoded movements from real neural activity, and blue lines represent decoded movements from synthetic neural activity. NMSE of decoded positions are shown in each panel. Decoded positions from DL models have relatively better NMSE compared to tuning and linear models.



Supplementary Figure 6: NMSE of positions decoded from recorded neural activity and those decoded from synthetic neural activity in the testing dataset. The red vertical line on each bar is the standard deviation of NMSE of repeated experiments. DL models achieve lower value than tuning models across three decoders. *** denoted $p < 0.001$, Wilcoxon rank-sum test.



Supplementary Figure 7: Decoded movements from real and synthetic neural activity with ReFIT-KF as decoder. In each panel, red lines are movements decoded from real neural data. Blue lines in each panel are movements decoded from synthetic neural activity of **a** PD, **b** PPVT, **c** GLM, **d** GLMh, **e** MLP, **f** MLP-h, and **g** RNN, respectively. The average NMSE of each encoder is shown in each panel. **a**, **b**, **c**, and **d** are decoded movements from PD, PPVT, GLM and GLMh. The movements do not reflect key motifs in original decodes and achieve poor NMSE. **e**, **f**, and **g** are decoded movements from DL models (MLP, MLP-h and RNN), showing more consistent synthetic decodes compared with the original decoding data. **h** Normalized mean squared error (NMSE) between positions decoded from recorded neural activity and those decoded from synthetic neural activity in closed-loop dataset. DL models achieve lower NMSE. *** denoted $p < 0.001$, Wilcoxon rank-sum test.

		PD-v	PPVT-v	GLM-v	GLMh-v	GLM-pv	GLMh-pv	MLP-v	MLP-pv	MLPh-pv	RNN-pv
PSTH	Pearson's r	0.0027	0.0021	0.0028	0.0029	0.0026	0.0021	0.0016	0.0060	0.0021	0.0028
	MSE(spikes ² /s)	0.0526	0.0468	0.0518	0.0332	0.0515	0.0328	0.1058	0.1486	0.0983	0.0477
PCA	Pearson's r	0.0029	0.0032	0.0052	0.0022	0.0015	0.0023	0.0019	0.0052	0.0024	0.0053
	MSE	0.0111	0.0022	0.0039	0.0033	0.0030	0.0024	0.0234	0.0743	0.0416	0.0279
jPCA	R^2_{skew}	0.0001	0.0001	0.0002	0.0059	0.0013	0.0056	0.0080	0.0201	0.0131	0.0143
	Skew Ratio	0.0008	0.0003	0.0005	0.0067	0.0015	0.0066	0.0143	0.0326	0.0283	0.0281
Decodes _{OL}	NMSE(OLE)	0.2423	0.0984	0.0568	0.0615	0.0601	0.0529	0.0433	0.0710	0.0697	0.0832
	NMSE(WF)	0.2480	0.1048	0.0778	0.0508	0.0756	0.0643	0.0486	0.0883	0.0420	0.0836
	NMSE(KF)	0.1881	0.0697	0.0641	0.0614	0.0403	0.0504	0.0664	0.0443	0.0536	0.0504
Decodes _{CL}	NMSE(ReFIT)	0.0826	0.0563	0.0764	0.0863	0.0577	0.0418	0.0619	0.0827	0.0972	0.0750

Table 1: Standard deviation of monkey J's results

		PD-v	PPVT-v	GLM-v	GLMh-v	GLM-pv	GLMh-pv	MLP-v	MLP-pv	MLPh-pv	RNN-pv
PSTH	Pearson's r	0.32	0.35	0.37	0.43	0.42	0.45	0.57	0.78	0.80	0.82
	MSE(spikes ² /s)	5.41	4.48	4.72	4.43	4.25	4.09	3.72	1.52	1.24	0.93
PCA	Pearson's r	0.22	0.25	0.31	0.44	0.39	0.46	0.61	0.82	0.85	0.86
	MSE	0.92	0.79	0.82	0.76	0.72	0.68	0.65	0.25	0.21	0.13
jPCA	R_{skew}^2	<0.01	<0.01	<0.01	0.12	0.02	0.11	0.08	0.13	0.15	0.21
	Skew Ratio	<0.01	<0.01	<0.01	0.13	0.02	0.12	0.12	0.24	0.29	0.38
Decodes _{OL}	NMSE(OLE)	1.82	1.55	1.41	1.19	0.95	0.84	1.27	0.58	0.52	0.57
	NMSE(WF)	1.65	1.43	1.45	1.27	1.29	1.11	1.31	0.97	0.81	0.87
	NMSE(KF)	1.64	1.40	1.24	1.09	0.88	0.75	1.14	0.54	0.46	0.51
Decodes _{CL}	NMSE(ReFIT)	2.18	2.44	2.44	2.26	1.87	1.78	2.35	1.51	1.24	1.62

Table 2: Summary of monkey L's results

		PD-v	PPVT-v	GLM-v	GLMh-v	GLM-pv	GLMh-pv	MLP-v	MLP-pv	MLPh-pv	RNN-pv
PSTH	Pearson's r	0.0025	0.0013	0.0021	0.0034	0.0033	0.0033	0.0026	0.0040	0.0042	0.0039
	MSE(spikes ² /s)	0.0756	0.0445	0.0574	0.0566	0.0502	0.0505	0.0627	0.0637	0.0626	0.0366
PCA	Pearson's r	0.0027	0.0036	0.0052	0.0033	0.0028	0.0047	0.0032	0.0046	0.0038	0.0056
	MSE	0.0055	0.0013	0.0043	0.0047	0.0042	0.0039	0.0044	0.0100	0.0101	0.0077
jPCA	R^2_{skew}	<0.0001	<0.0001	<0.0001	0.0063	0.0000	0.0054	0.0045	0.0095	0.0154	0.0121
	Skew Ratio	<0.0001	<0.0001	<0.0001	0.0069	0.0000	0.0059	0.0071	0.0174	0.0299	0.0245
Decodes _{OL}	NMSE(OLE)	0.1466	0.1045	0.0834	0.0608	0.0586	0.0595	0.0632	0.0453	0.0497	0.0554
	NMSE(WF)	0.1409	0.0937	0.1126	0.0838	0.0832	0.0751	0.0781	0.0959	0.0834	0.0957
	NMSE(KF)	0.1002	0.0929	0.0514	0.0525	0.0519	0.0436	0.0481	0.0380	0.0302	0.0490
Decodes _{CL}	NMSE(ReFIT)	0.0579	0.0726	0.0872	0.0756	0.0538	0.0774	0.0863	0.0715	0.0691	0.0838

Table 3: Standard deviation of monkey L's results