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Apparent Radiation Characteristics of Semitransparent Media Containing Gas Bubbles

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Abstract

In many materials processing and manufacturing situations such as steel, aluminum, ceramics and glass, gas bubbles can form in liquid and solid phases. The presence of such bubbles affects the thermophysical properties and radiation characteristics of the two-phase system and hence the transport phenomena. This paper presents a general formulation of the radiation characteristics of semitransparent media containing gas bubbles (with bubbles which have radius much larger than the wavelength of radiation). Sample calculations for the spectral absorption and extinction coefficients and single scattering albedo of soda-lime silicate glass containing bubbles are discussed. Particular attention is paid to the effect of the volumetric void fraction and the bubble size distribution. Results clearly show that the presence of bubbles strongly affects the radiation characteristics of the semitransparent media containing entrapped gas bubbles, particularly if bubbles, void fractions, and spectral absorption coefficient of the continuous phase are small.

1 Introduction

In many materials processing and manufacturing situations such as steel, aluminum, ceramics and glass, gas bubbles can form in liquid and solid phases. The presence of such bubbles affects the thermophysical properties and radiation characteristics of the two-phase system and hence the transport phenomena. In glass melting furnaces, for example, a large number of bubbles are formed by chemical reactions during melting of the batch and the thermal decomposition of refining agents. Bubbles that are large enough rise to the surface of the glass melt while small bubbles are trapped in the flowing molten glass. The quality of the glass produced is affected by the flow pattern and the temperature of the glass melt. The analyses reported in the literature have neglected the presence of numerous gas bubbles of different sizes in the melt that may affect the radiative heat transfer (Viskanta, 1994). The bubbles affect the radiation characteristics of the glass melt since the radiation is scattered by the bubbles. Approximate and rigorous treatements of radiative transfer in glass require spectral radiation properties of glass melt which contains gas bubbles. Therefore, it is of particular interest to understand the effect of bubbles on the radiation characteristics of the molten glass in order to better predict radiation heat transfer in glass melting furnaces, and improve the glass quality and the energy efficiency of the processes.

The objective of this work is to explore the effect of the bubble radius, the void fraction and the depth of the glass layer on radiation characteristics of glass containing bubbles.

2 Analysis

Consider heat transfer within an horizontal layer of continuous condensed phase containing bubbles as shown in Figure 1. We further assume that the continuous phase is a solid or a slowly moving liquid and is essentially isothermal. Theconduction and convection heat transfer can be safely neglected in comparison to heat transfer by radiation. When gas bubbles are moving with the liquid phase, a time-averaged void fraction and bubble size distribution should be used. In addition, we assume the following: (1) all bubbles are spherical, (2) the scattering of a single bubble is not affected by the presence of its neighbors (independent scattering); and (3) the radiation field within the liquid layer is incoherent (i.e., scattering centers are randomly distributed with zero-phase correlation). Then, radiative transfer within an absorbing, emitting, and independently scattering medium is governed by the so-called radiative transfer equation (RTE) [see (Modest, 1993)]. Solving the RTE requires the knowledge of the effective absorption κ_λ, scattering σ_λ , and extinction β_λ $(= \kappa_\lambda + \sigma_\lambda)$ coefficients together with the scattering phase function and the single scattering albedo defined as $\omega_\lambda = \sigma_\lambda/(\kappa_\lambda + \sigma_\lambda) = \sigma_\lambda/\beta_\lambda$.

Let $m^d_\lambda = n^d_\lambda - ik^d_\lambda$ and $m^c_\lambda = n^c_\lambda - ik^c_\lambda$ be the spectral complex indices of refraction of the dispersed phase (i.e., gas bubbles), and of the continuous phase, respectively. The spectral and apparent radiation characteristics of glass containing monodispersed bubbles can be predicted from previously derived formulas (Fedorov and Viskanta, 2000b).

2.1 Prediction of Spectral Radiation Characteristics for Monodispersed Bubbles

Figure 1 depicts a sketch of a plane parallel layer of the absorbing and scattering medium 2 (medium of interest). The top surface of the layer is exposed to a medium 1 (say, combustion gases), and its bottom surface is in contact with a medium 3 (say, dense molten glass). The incident radiation is assumed to be diffuse and the average temperature of the layer is assumed to be much smaller than that of the radiation source, thus the emission of radiation by the layer can be neglected. We assume here that all the bubbles entrapped in the glass layer have a uniform radius a. Then, the effective extinction coefficients (due to absorption and scattering) for the layer can be expressed as (Fedorov and Viskanta, 2000b)

$$\kappa_\lambda = \kappa^c_\lambda - \pi[Q^c_{abs}(a) - Q^d_{abs}(a)]a^2 N_T \quad (1)$$

$$\sigma_\lambda = \pi Q^d_{sca}(a) a^2 N_T \quad (2)$$

$$\beta_\lambda = (\kappa_\lambda + \sigma_\lambda) = \kappa^c_\lambda - \pi[Q^c_{abs}(a) - Q^d_{ext}(a)]a^2 N_T \quad (3)$$

where $Q_{abs}(a)$, $Q_{sca}(a)$, $Q_{ext}(a)$ denote the absorption, scattering, and extinction efficiency factors, respectively, for a sphere of radius a, while the superscripts "d" and "c" refers to the dispersed and the continuous phase, respectively. The total number of bubbles per unit volume is N_T and can be expressed as a function of the average void fraction f_v in the layer and densities: $N_T = 3f_v/(4\pi a^3)$. Moreover, for independent scattering, the phase function in a cloud of uniform bubbles is the same for each particle, it is also the same for a bubble cloud, i.e., $\Phi_\lambda(\Theta) = \phi(a, \Theta)$. Note that the absorption coefficient of the continuous phase κ^c_λ in Equation (1) can be calculated from the imaginary part (k^c_λ) of the complex index of refraction (m^c_λ) as $\kappa^c_\lambda = 4\pi\eta_0 k^c_\lambda$, where $\eta_0 = \nu/c_0 = 1/(n^c_\lambda \lambda)$ is the wavenumber of the wave with a frequency ν and phase velocity equal to a speed of light in vacuum c_0.

Evaluation of the efficiency factors Q_{abs}, Q_{sca} and Q_{ext} from the Mie theory is computer resources intensive. In a multidimensional and spectral radiative transfer analysis this type of approach becomes impratical. Therefore, it is desirable to have a simple approximation for the efficiency factors. The changes in the scattering pattern due to changes in the bubble size should be accounted for in the prediction of the radiative properties of the liquid layer containing bubbles. For spheres with index of refraction close to 1, the $\rho - \chi$ domain can be divided into two limiting cases (van de Hulst, 1957):

- The Rayleigh-Gans scattering domain corresponds to a near-dielectric sphere with (1) $k \approx 0$, (2) a refractive index of refraction close to unity, i.e., $|m - 1| \ll 1$, and such that (3) the phase lag suffered by the central ray that passes through the sphere along a diameter is small, i.e., $\rho = 2\chi|m - 1| \ll 1$. Then, the reflectivity is negligible and the radiation passes through the sphere unattenuated and unrefracted (van de Hulst, 1957). The Rayleigh-Gans scattering domain can itself be divided into two limiting cases, namely, $\chi \to 0$ (Rayleigh scattering) and $\chi \to \infty$ ("intermediate regime") (van de Hulst, 1957).
- The anomalous-diffraction domain is characterized by $\chi \to \infty$ and $m \to 1$ corresponding to a straight transmission and subsequent diffraction according to Huygens' principle (van de Hulst, 1957). The anomalous-diffraction domain can also be divided into two limiting cases, namely, $\rho \to 0$ ("intermediate") and $\rho \to \infty$ ("geometrical optics + diffraction").

Note that the Rayleigh-Gans domain and the anomalous diffraction domain overlap in the so-called "intermediate regime".

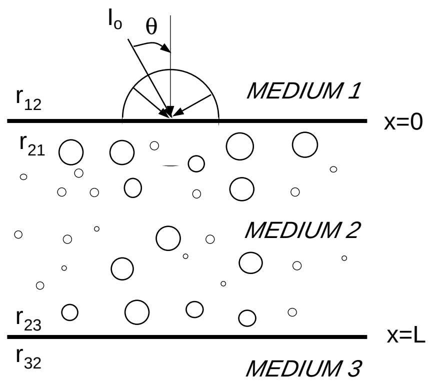

Figure 1: Schematic of the idealized liquid layer containing bubbles and the coordinate system.

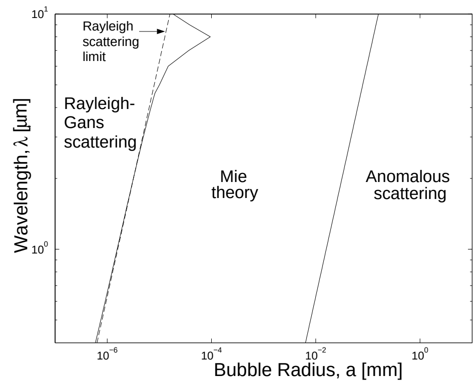

Figure 2: Map of the scattering theories and approximations used for determining the extinction efficiency factors of the soda-lime silicate containing gas bubbles, i.e, Q_{abs}, Q_{sca}, and Q_{ext}.

The present work is concerned with relatively large bubbles and wavelengths between 0.4 μm and 10 μm for which the anomalous scattering is valid. In this region of the $\rho - \chi$ domain, the gas bubbles are relatively weak absorbers of radiation and mostly act as the strong radiation scatterers. The approximate analytical expressions for absorption and scattering efficiency factors and a weakly absorbing sphere of arbitrary size can be used as derived by van de Hulst (1957, p. 179) and reported by Fedorov and Viskanta (2000b). Because of the assumptions of the van de Hulst's theory, the expression overestimates the extinction factor for small spheres and underestimates it for larger spheres. To correct this, Deirmendjian (1969) proposed to use a correction factor $(1 + D_i)$. The approach was remarkably successful in improving accuracy of extinction coefficient and the specific expressions for D_i can be found in the literature (Deirmendjian, 1969, pp. 29-30). Comparison of the absorption and scattering efficiencies generated using the approximate expressions with numerical results obtained from the exact Mie theory (Deirmendjian, 1969, pp. 30-32), have demonstrated the power of this approach, particularly when precise directional scattering pattern and polarizing properties are not desired. Specifically, the results obtained via the corrected van de Hulst's formula $(1 + D_i) \times Q_{ext}$ are accurate within about $\pm 0.05 Q_{ext}$ for a wide range of sphere sizes and indices of refraction.

2.2 Model Validity for Glass Containing Bubbles

In defining the limiting cases of the Mie theory for which simple analytical solutions are known, we used the qualitative criteria $\rho \ll 1$ for Rayleigh-Gans scattering and $\chi \gg 1$ for anomalous diffraction. For our particular application the different scattering regimes are delimited arbitrarily as follows,

- Anomalous scattering approximation is assumed to be valid for $\chi \geq 100$. This condition leads to $4a \geq 100\lambda/2\pi$.
- The Rayleigh-Gans scattering approximation is assumed to be valid for $\rho \leq 0.01$. For the gas bubbles, this condition is equivalent to $a \leq \lambda / \left[400\pi\sqrt{(n_\lambda - 1)^2 + (k_\lambda)^2}\right]$. The index of refraction of soda-lime silicate glass depends strongly on the wavelength and must be accounted for in defining the scattering domains. The Rayleigh-Gans scattering approximation can be made when the condition expressed by the above equation is valid for both the dispersed and continuous phase.
- A subdomain of the Rayleigh-Gans scattering approximation is the Rayleigh scattering which is assumed to be valid when $\rho \leq 0.01$ and $\chi \leq 0.01$. For the gas bubbles and glass spheres, these conditions are expressed by the above equation and $a \leq \lambda/200\pi$.

One needs to consider the absorption efficiency factor for both gas bubbles and the corresponding glass spheres, and the scattering efficiency factor and the scattering phase function for the gas bubbles in order to predict the effective radiation characteristics of the glass layer containing bubbles. ¿From Figure 2, one can conclude that for bubbles with radius larger than 0.1 mm the radiation characteristics of the glass layer can be predicted from the anomalous diffraction theory. For bubbles less than 1 nm in diameter, the same radiation characteristics of the glass layer can be estimated from the Rayleigh-Gans scattering theory. However, for bubbles having radii between 1 nm and 0.1 mm and/or if the void fraction is larger than 0.006 the use of the Mie theory and/or the consideration of dependent scattering is required for wavelengths between 0.4 μm and 10 μm. This makes the computation of the apparent absorptance, reflectance and transmittance of the layer considerably more involved. Therefore, only large bubbles for which the anomalous diffraction theory and the independent scattering assumption (Tien and Drolen, 1987) are valid will be considered further.

Tien and Drolen (1987) presented a scattering regime map which uses the size parameter ($\chi = 2\pi a/\lambda$) and the volume fraction (f_v) as the coordinate axis. They showed that the dependent scattering effects may be ignored as long as $f_v < 0.006$ or $c/\lambda < 0.5$. Assuming a cubic lattice of bubbles of pitch p, the condition $c/\lambda < 0.5$ can be expressed in terms of the void fraction as $f_v < (32\pi/3)(a/\lambda) = 16\chi/3$. This suggests that for bubbles larger than 1 μm in diameter, dependent scattering can be safely neglected.

The objective of this work is to predict the radiation characteristics of glass containing spherical gas bubbles with different bubble size distributions and to gain understanding of their importance on radiative transfer in glass. It can be shown (Franses, 2000) that bubbles are spherical if their radius a is small compared to the capillary length l_c ($a \ll l_c$) where the capillary length for gas bubbles surrounded by liquid is defined as $l_c = \sqrt{2\sigma/[(\rho^c - \rho^d)g]}$. Here, σ is the surface tension (=300 mN/m), and ρ^c [=2350 kg/m^3 at around 1400 K (Laimbock, 1998)] and ρ^d (=1.2 kg/m^3) are the densities of the molten soda-lime silicate glass and the air, respectively. Hence, for soda-lime silicate glass the capillary length l_c is about 4 mm. We assumed that bubbles are spherical for bubble radii up to 1 mm.

2.3 Radiation Characteristics of the Glass Containing Bubbles

In this section we first discuss the effect of the bubble radius and the void fraction on the radiation characteritics of soda-lime silicate glass containing bubbles of uniform size. As a concrete example, the spectral absorption and extinction coefficients as well as the single scattering albedo have been predicted for clear soda-lime silicate glass containing monodispersed bubbles for a volumetric void fractions of 0.2. Three different radii a were considered, 0.2 mm, 0.8 mm, and 1.6 mm.

¿From Figure 3 one can see that the presence of bubbles reduces the absorption coefficient in the spectral region of 0.4 to 4.5 μm where the absorption coefficient of the glass is relatively small. In this same region, the extinction coefficient is strongly affected by the presence and the size of the bubbles. The scattering is particularly important for smaller bubbles, and the single scattering albedo is close to unity. This indicates that the radiative heat transfer is dominated by scattering rather than by absorption for $0.4 < \lambda < 4.5$ μm. In the spectral region 4.5 to 10 μm, however, the absorption coefficient of the dense glass is large and the presence and the size of the bubbles have little effect of the effective absorption coefficient of the glass layer. In other words, the scattering coefficient is negligible compared with the absorption coefficient and the radiative heat transfer is dominated by absorption.
Moreover, the absorption coefficient decreases significanlty as the void fraction increases and can be reduced by up to one order of magnitude for void fractions f_v varying from 0.2 to 0.6 (results not shown). In contrast, the extinction coefficient and the single scattering albedo increase as the void fraction or the number of bubbles increase. This can be explained by the fact increasing the void fraction increases the number of scatterers, while the absorption by the two-phase mixture decreases.

2.4 Apparent Radiation Characteristics of the Layer: Accounting for Reflecting Boundaries

Apparent reflectances, transmittances and absorptances of a glass layers containing large bubbles have been calculated using the previously developed theoretical methodology (Fedorov and Viskanta, 2000b) are reported

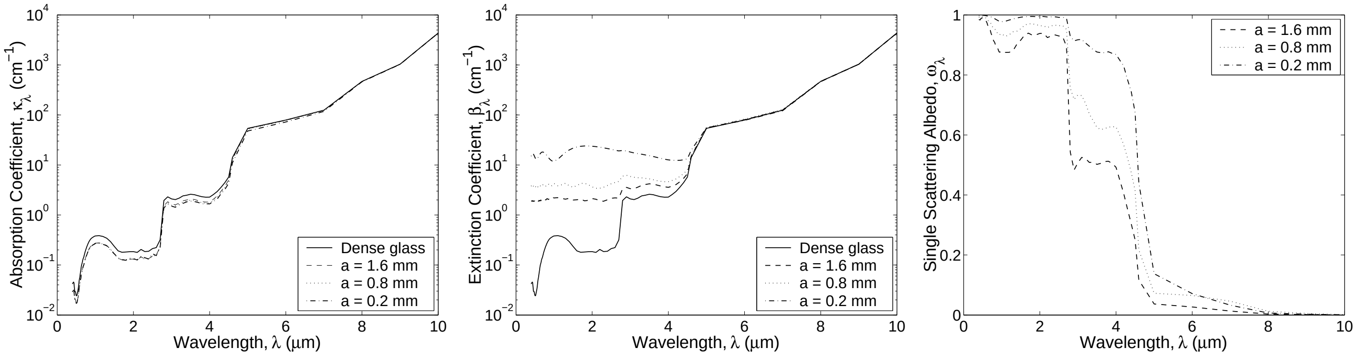

Figure 3: Effect of bubble radius on the spectral absorption and extinction coefficients and single scattering albedo for soda-lime silicate glass with $f_v = 0.2$.

and discussed. All numerical calculations have been performed for the case of the isotropic incidence only. The reader is referred to the literature for collimated incidence (Fedorov and Viskanta, 2000b). Furthermore, since for independent scattering, the scattering phase function of the layer is the same as that of a single bubble and since large bubbles tend to feature a forward scattering phase function, only the forward scattering phase function is considered in assessing the effect of the glass layer thickness.

Figure 4 shows the apparent reflectance ($R^\star$), transmittance ($T^\star$), and absorptance ($A^\star$) as a function of the void fraction, respectively, for a 5 cm glass layer containing bubbles 1 mm in radius. As expected, with an increase in the amount of gas contained in the layer, the apparent reflectance increases, whereas the transmittance and the absorptance decrease until they reach their asymptotic limits of about 80%, 12% and 8%, respectively. These values should be compared to those when no bubbles are present, i.e., approximately 30%, 70% and 0%, respectively. The asymptotic value of the apparent reflectance is significantly large for 1 mm bubbles and decreases as the bubble radius increases. It is interesting to note that the apparent characteristics of the glass layer are strongly affected by even a small amount of gas bubbles retained in the glass. However, beyond void fractions of 0.2 to 0.3, the apparent radiation characteristics do not vary significantly. This findings can be explained by the fact that for small void fractions the scattering pattern of the layer is strongly affected by the bubbles, and the radiation tends to be back scattered by the bubbles, thus the apparent reflectance increases. Similarly, as more and more radiation is scattered back, less is transmitted through the layer and the apparent transmittance decreases accordingly. For a larger void fraction, the scattering pattern of the layer is established and the apparent characteristics reach their asymptotic values. The absorptance increases sharply with increasing void fraction up to 0.3 and then varies little for higher void fractions. This behavior can be explained by the fact that as the radiation is scattered by the bubbles its path within the glass phase increases; therefore, the absorptance of the glass layer increases. However, for larger void fractions, the radiation path in the layer increases but the rays "travel" mainly through the gas phase. Hence, the increase in absorption due to longer paths in the glass layer is compensated by the presence of a gas bubbles that do not absorb radiation.

Figure 5 shows the variations of the effect of the glass layer thickness on the apparent radiation characteristics as functions of the glass layer thicknesses containing 1 mm radius bubbles. As expected and as previously noted by Fedorov and Viskanta (2000a), with an increase in the thickness of the layer, the apparent transmittance decreases exponentially until it reaches a minimum, whereas the reflectance and absorptance of the 5 cm glass layer gradually increase until they reach asymptotic limits of about 72 % and 28 %, respectively. As the glass layer thickness increases, the characteristics reach their asymptotic values for smaller void fractions. In other words, the thicker the glass layer, the more sensitive it is to changes in small values of the void fraction. Figure 5 indicates that even a small number of bubbles can have a strong effect on the apparent characteristics of tanks deeper than 0.5 m.

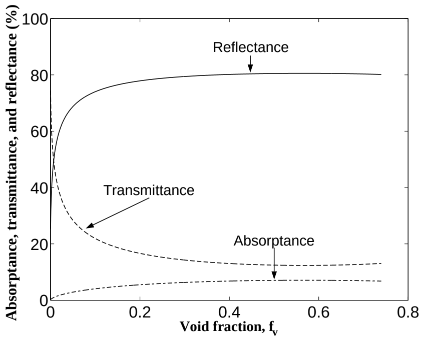

Figure 4: Apparent reflectance, transmittance, and absorbance of a 5 cm thick glass layer containing gas bubbles 1 mm in radius at a wavelength of 2 μm.

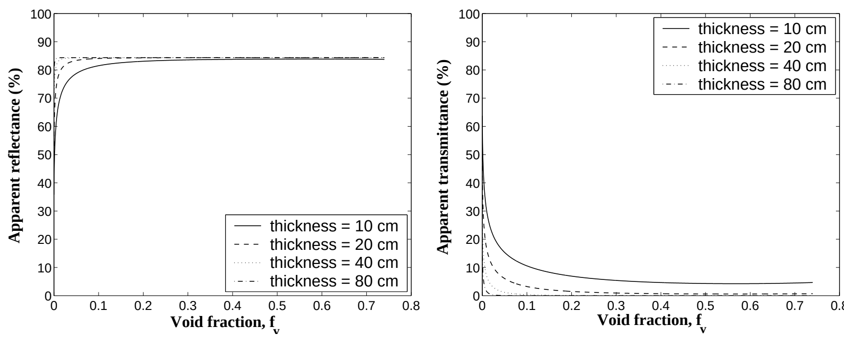

Figure 5: Effect of the glass layer thickness on the apparent reflectance and transmittance at wavelength of 2 μm for soda-lime silicate glass containing bubbles 1 mm in radius.

3 Concluding remarks

An analysis of radiative transfer in a semitransparent glass layer containing gas bubbles has been presented. The results of sample calculations performed lead to the following conclusions:

- For bubbles less than 10 μm in diameter and void fractions larger than 0.006, the Mie theory should be used and/or considerations of dependent scattering are required.
- For bubbles larger than 0.1 mm in radius the analysis developed for glass foams by Fedorov and Viskanta (2000a,b) can be extended over the entire range of void fractions (from 0 to 0.74).
- Even small void fractions affect the apparent characteristics of the glass layer containing large bubbles. The effect of the void fraction is even more significant for large glass thicknesses. Therefore, in modeling the radiative heat transfer in glass melting furnaces one should consider the effects of gas bubbles on the radiation characteristics of the molten glass.
- Beyond a small critical void fraction (about 0.3 for 5 cm thick glass containing bubbles of 1 mm in radius), the apparent characteristics do not change significantly.
- Unlike the apparent absorptance, the apparent reflectance and transmittance are very sensitive to the scattering phase function. A larger reflectance and a smaller transmittance are found for isotropic scattering than for forward scattering.

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