SOME RECENT RESULTS IN MOVING LOAD PROBLEMS WITH APPLICATION TO HIGHWAY BRIDGES

A. V. Pesterev[1], L. A. Bergman[2], C. A. Tan[3], T.-C. Tsao[4], and B. Yang[5]

[1] *Institute for Systems Analysis, Russian Academy of Sciences, Moscow, pesterev@isa.ru*
[2] *Aeronautical and Astronautical Engineering Department, University of Illinois, Urbana, lbergman@uiuc.edu*
[3] *Department of Mechanical Engineering, Wayne State University, Detroit, tan@wayne.edu*
[4] *Mechanical and Aerospace Engineering Department, University of California, ttsao@seas.ucla.edu*
[5] *Department of Mechanical Engineering, University of Southern California, bingen@almaak.usc.edu*

ABSTRACT

In this paper, some recent results obtained for various aspects of the moving load problem are discussed. While encompassing a range of problems involving discrete subsystems traversing a primary distributed parameter system, a simple vehicle crossing a bridge is used herein to demonstrate various effects and their relative importance.

1. INTRODUCTION

Forces and/or subsystems moving along a distributed structural system is a common engineering problem. The critical issues are the accurate prediction of displacements, velocities, accelerations, and stresses throughout the system and characterization of the dynamic interactions developed between all of the interacting subsystems. Interest in this class of problems appears to have originated with the pioneering work of Willis and Stokes in their analyses of railway bridges during the mid-19th century, and this particular application is still pertinent today in the design of roadbeds and bridges traversed by high-speed trains. Similar problems are also encountered in design of elevated roadways and bridges carrying heavy vehicles, parking garages and aircraft carriers, ballistic systems such as railguns, high-speed precision machining, magnetic disk drives, cables supporting humans and materials, disk brakes, systems subjected to pressure waves, and so forth. An important characteristic of these systems is the absence of a steady-state solution to the response problem. Thus, the solution is fully defined by the transient responses of the primary, or continuous, subsystem and the secondary, or discrete, subsystems, and many well-known classical methods of analysis are no longer applicable. Moreover, when the dynamic interactions between subsystems are important, the solution must be obtained by simultaneously solving the resulting coupled system of partial and ordinary differential equations in time. In this paper, some recent results obtained for various aspects of this problem are discussed. A simple vehicle traversing a bridge is used throughout in order to demonstrate various effects and their relative importance.

2. THE MOVING LOAD PROBLEM AND A GENERAL METHOD FOR ITS SOLUTION

First, we consider a comprehensive mathematical statement of the problem of a single vehicle traversing a bridge and discuss a general approach to its solution. Let the bridge be modeled as a continuum, and $w(x,t)$ be the displacement of a continuum point x at time t. The equation governing vibration of the continuum can be written in the operator form

$$\hat{A}_c w(x,t) = F_c(x,t),\ x \in \Omega, \tag{1}$$

where $\hat{A}_c$ is a partial differential operator determined by an accepted model of bridge and F_c is a force acting on the continuum. Note that boundary conditions are included in the definition of the operator $\hat{A}_c$.

Let the vehicle be modeled as a finite-dimensional subsystem, and $z(t)$ be a vector of its coordinates. The equation governing vibration of the vehicle can be expressed as

$$\hat{A}_v z(t) = F_v(t),\ z(t) \in R^n, \tag{2}$$

where $\hat{A}_v$ is an ordinary differential operator, n is the number of DOFs of the finite-dimensional subsystem, and $F_v(t) \in R^n$ is a vector of forces acting on the finite-dimensional system.

Solutions to equations (1) and (2) are well known to be

$$\hat{A}_c^{-1} F_c(x,t) = \int_0^t \int_\Omega G_c(x,\xi,t-\tau) F_c(\xi,\tau)\, d\xi\, d\tau, \tag{3}$$

$$\hat{A}_v^{-1} F_v(t) = \int_0^t G_v(t-\tau) F_v(\tau)\, d\tau, \qquad (4)$$

where $G_c(x,\xi,t)$ and $G_v(t)$ are the Green's functions of the distributed-parameter and finite-dimensional subsystems, respectively, which are calculated either numerically or, if the modal data are available, in terms of the modal series.

Now, let the subsystems interact, and let the interaction be described by a linear operator $\hat{\Delta}A$. We will write it in a factored form, $\hat{\Delta}A = \hat{\Psi}\hat{\Phi}$ (we will not discuss the ways of the decomposition of this operator, since it will draw us away from the main subject, and note only that such a representation is always possible, since, e.g., we can consider a trivial decomposition: $\hat{\Psi} = I$ (I is the identity operator), $\hat{\Phi} = \hat{\Delta}A$, and is not unique). Introduce the notation

$$\hat{A} = \begin{bmatrix} \hat{A}_c & 0 \\ 0 & \hat{A}_v \end{bmatrix}, X = \begin{Bmatrix} w(x,t) \\ z(t) \end{Bmatrix}, F = \begin{Bmatrix} F_c(x,t) \\ F_v(t) \end{Bmatrix}.$$

Then, assuming that $F_c(x,t)$ and $F_v(t)$ are external forces, the equation governing coupled vibration of the linear system can be written as

$$\hat{\tilde{A}} X \equiv (\hat{A} + \hat{\Psi}\hat{\Phi}) X = F, \qquad (5)$$

where $\hat{\tilde{A}}$ denotes the operator governing coupled vibration of the interacting subsystems.

To find a solution X of equation (5), we need the inverse of the operator $\hat{\tilde{A}}$. According to the theorem on the perturbation of an invertible operator [1–3], the operator $\hat{\tilde{A}} = \hat{A} + \hat{\Psi}\hat{\Phi}$ is invertible if and only if the characteristic operator

$$\hat{\chi} = I + \hat{\Phi}\hat{A}^{-1}\hat{\Psi} \qquad (6)$$

is invertible. If the characteristic operator is invertible, then

$$\hat{\tilde{A}}^{-1} = \hat{A}^{-1} - \hat{A}^{-1}\hat{\Psi}\hat{\chi}^{-1}\hat{\Phi}\hat{A}^{-1}. \qquad (7)$$

It follows from physical considerations that the original problem of a moving vehicle always has a solution. Hence, both $\hat{\tilde{A}}$ and $\hat{\chi}$ are invertible. By applying the RHS of equation (7) to the vector of external forces, we get

$$X = \hat{A}^{-1} F - \hat{A}^{-1}\hat{\Psi} y, \qquad (8)$$

where

$$y = \hat{\chi}^{-1}\hat{\Phi}\hat{A}^{-1} F. \qquad (9)$$

Equation (8) can easily be interpreted. The solution X is seen to be the vibration of two non-interacting subsystems subject to the two sets of forces: external forces F and the interaction forces $\hat{\Psi}y$. Rewriting (9) as

$$\hat{\chi} y = \hat{\Phi}\hat{A}^{-1} F$$

and taking into account equation (6), we find that y is a solution to the operator equation

$$y + (\hat{\Phi}\hat{A}^{-1}\hat{\Psi}) y = \hat{\Phi}\hat{A}^{-1} F, \qquad (10)$$

which is usually found in a simpler way than the solution of the original equation governed by the differential operator $\hat{\tilde{A}}$.

As can be seen, the problem of a moving load is decomposed into two problems: (i) finding the vector $y(t)$ (the components of which are, up to scaling coefficients, forces of interaction between the subsystems), which is a solution to the Volterra equation (9) (or (10)), and (ii) finding the response $X(x,t)$ of the subsystems due to given external and interaction forces by equation (8).

Let us establish the relationship between the vectors $X(x,t)$ and $y(t)$. Acting on the both sides of equation (8) by the operator $\hat{\Phi}$, we get

$$\hat{\Phi} X = \hat{\Phi}\hat{A}^{-1} F - \hat{\Phi}\hat{A}^{-1}\hat{\Psi} y.$$

Comparing this equation with (10), we find that

$$y(t) = \hat{\Phi} X(x,t). \qquad (11)$$

Note that, in the preceding discussion, we considered a general case of interacting subsystems to emphasize the fact that the approach is applicable to any problem involving linear vibration of coupled interacting systems. Considering different problems, we arrive at different interaction operators and, thus, to different operator equations (10). This approach was applied earlier to the problem of vibration of a continuous system carrying finite-dimensional subsystems at fixed points [2–5]. In that case, the operators $\hat{\Phi}$ and $\hat{\Psi}$ do not depend on time, and the characteristic operator $\hat{\chi}$ is a matrix of size equal to the rank of the interaction.

The extension of this approach to the moving load problem was first discussed in [6], where it was applied to the problem of vibration of a beam traversed by an SDOF conservative oscillator. Later, it was applied in [7] to the case of an arbitrary moving finite-dimensional subsystem. In this case, the coordinates of points where the subsystems interact are functions of time; i.e., the interaction operator and, hence, $\hat{\Phi}$ and $\hat{\Psi}$ are time-dependent operators. In the case of an SDOF moving oscillator, the characteristic operator is a Volterra integral equation of the second kind [6]; in the general case of an MDOF vehicle model interacting with the continuum at m points, we arrive at a system of m Volterra equations in the m-dimensional vector $y(t)$.

To illustrate the preceding discussion, we present the factorization of the interaction operator and the resulting Volterra equation for the case of a conservative SDOF oscillator of mass m_0 moving with constant speed v along a one-dimensional continuum (beam). As shown in [6], the interaction operator can be represented as the product of the column operator $\hat{\Psi}$ and the row operator $\hat{\Phi}$ given by

$$\hat{\Psi} = \left\{ \begin{array}{c} \delta(x - vt) \\ -1 \end{array} \right\}, \; \hat{\Phi} = k[\hat{\pi}_x(vt), -1],$$

where k is the stiffness of the spring connecting the moving mass to the continuum, $\delta(x)$ is the Dirac delta-function, and $\hat{\pi}_x(vt)$ is the operator substituting vt for x, $\hat{\pi}_x(vt)f(x) = f(vt)$. Taking these and equations (3), (4), (6) into account, we find that the characteristic operator is the the integral Volterra operator

$$\hat{\chi}y(t) \equiv y(t)+k\int_0^t [G_c(vt, v\tau, t-\tau)+G_v(t-\tau)]y(\tau)\,d\tau,$$

where $G_v(t)$ is the Green's function of the mass m_0, $G_v(t-\tau) = (t-\tau)/m_0$. The RHS of the integral equation (10) is determined by the external forces acting on both subsystems. Note that, in this approach, the weight of the moving subsystem is assumed to be an external force, such that the RHS of (10) is always nonzero.

The advantage of the approach based on the Volterra equations is that it allows us to get the solution in a unique, general, and concise form. Given that the Green's operators of the continuum and the moving subsystem(s), and, hence, the inverse operators $\hat{A}_c^{-1}$ and $\hat{A}_v^{-1}$, are known (can be efficiently calculated), the solution of the moving vehicle problem is given by equations (8), (9).

An equivalent approach, which avoids solving the Volterra equations, exists. In practice, the Green's functions are usually calculated in terms of modal series. Expanding the responses of the continuum and finite-dimensional system in terms of their eigenfunctions and using N continuum eigenfunctions to approximate the continuum response, we can write X in the form $X = \phi(x)q(t)$, where $\phi(x)$ denotes a block matrix containing eigenfunctions of both continuum and finite-dimensional subsystem, and $q(t)$ is an $(N + n)$-vector of modal coordinates. By virtue of (11), the vector $y(t)$ is also expressed in terms of $q(t)$ as $y = \hat{\Phi}\phi q$. Substituting this expression into equation (10) or (8) (both equations should result in the same equations in q) and using the modal series representation for the Green's functions, we get a system of $N + n$ integral equations in the modal coordinates $q_j(t), j = 1, \ldots, N + n$. It can be shown that, by differentiating the equations obtained, we get a set of ODEs in q_j. We will not derive the set of ODEs for the general case here; however, they were given in [7].

Summarizing the results of this section, we see that:

(i) Any moving vehicle problem can be solved by applying the unique method described above for arbitrary bridge and vehicle models if the Green's operators of both subsystems can be efficiently calculated (modal data are available). The problem of finding the response of the system reduces to solving a set of Volterra equations in the interaction forces between the bridge and vehicle. The latter problem can be reduced to solving a set of ODEs in the modal coordinates of the expansion of the system response in terms of the bridge and vehicle eigenfunctions.

(ii) In the framework of the above approach, there is no fundamental difference whether the moving vehicle is modeled as an SDOF oscillator or an MDOF system. Both problems are formulated and solved in the same way, with the difference being only in the computational complexity ($N + 1$ versus $N + n$ second-order equations).

3. MOVING LOAD MODELS

In this section, we describe various models for moving loads employed in the literature for analyzing the vibration of bridges traversed by vehicles and discuss limits of their applicability. Our goal is to try to understand which parameters of the moving vehicle problem are most important from the standpoint of bridge vibration, and in which cases we should use more complicated, rather than simple, vehicle models. The latter issue is very important for the following reasons: Although, as noted previously, the solution of the moving vehicle model can be obtained by applying the same method independent of vehicle model considered, the computational complexity increases with the growth in dimension of the MDOF system used for modeling the vehicle. Moreover, the use of complicated vehicle models considerably complicates the subsequent analysis of the results of numerical experiments in view of the large number of parameters involved.

Rewrite equation (1) governing vibration of the continuum due to a moving load in the form

$$\hat{A}_c w(x,t) = f_c(x,t) + f_{ext}(x,t), \tag{12}$$

where $f_c(x,t)$ is a vector of forces acting on the continuum from the moving load and $f_{ext}(x,t)$ are external forces acting on the continuum. Here, though, we assume that the vehicle weight is included in the force $f_c(x,t)$. In addition to this equation, we must specify the law of motion of the load along the continuum; e.g., indicate the function $x = \zeta(t)$ showing position of the load on the continuum at time t (if there are multiple points where the forces from the moving vehicle are applied, $\zeta(t)$ is a vector). Since we are interested in the effect of the moving load, rather than other factors, on the bridge vibration, we assume in the following that $f_{ext}(x,t) = 0$. We also assume that the vehicle moves with a constant speed v and enters the bridge at $t = 0$, such that $\zeta(t) = vt$.

The various moving load models are defined by different forms of the function $f_c(x,t)$ on the RHS of (12).

1. In the *moving force model,* the force is assumed to be constant and equal to the vehicle weight,

$$f_c(x,t) = P\delta(x - vt),$$

where P is the vehicle weight. This is the simplest vehicle model, since we have only two parameters: vehicle weight and speed. The bridge and vehicle do not interact, and we have N equations in the time-dependent coefficients of the expansion of the continuum response which is in terms of the continuum eigenfunctions.

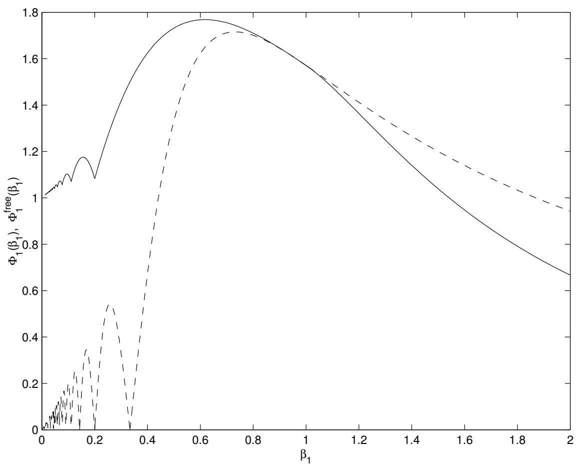

Figure 1: Dependence of the maximum deflection of the dimensionless simply supported beam, subject to a moving force, on the dimensionless speed of the traveling force.

Figure 1 [8] shows the ratio of the maximum deflection of the dimensionless simply supported beam to the maximum static deflection of the beam due to the same force applied to the beam midpoint vs. speed of the traveling force, where the dashed line shows the amplitude of the beam free vibration after the force leaves the beam. The value $\beta_1 = 1$ of the dimensionless speed parameter corresponds to the "critical speed" v_1 for which the force traverses the beam for the time equal to the half-period of the fundamental beam vibration, $T_p = T_1/2$. As can be seen, the maximum deflection of the beam is about 70% more than the static deflection and is attained for $v \approx 0.6v_1$. However, this speed is unlikely to be seen in bridge applications. Our analysis of experimental results reported in the literature shows that $T_p > 5(T_1/2)$, such that the upper bound for β_1 is less than 0.2. As can be seen from Fig. 1, the maximum dynamic deflection of the beam in this speed range does not exceed 10–15% of the static deflection.

2. The *moving mass model* accounts for the inertial effect of the moving load. Formally, the moving mass model does not fall into the category discussed previously, since there is no real interaction between the bridge and the mass. However, we can apply these methods by modeling the moving mass as an oscillator with large spring stiffness. The force is given by

$$f_c(x,t) = \left(P - m\frac{d^2w(vt,t)}{dt^2}\right)\delta(x - vt).$$

The difference between solutions of the moving force and moving mass models grows with increasing speed. For speeds of interest in bridge applications, which are classified as low, the models yield nearly identical results.

3. In the *moving oscillator model,* the mass m_0 is attached to the continuum through a spring and dashpot,

$$f_c(x,t) = [P + k(z(t) - w(vt,t)) + c(\dot{z}(t) - \dot{w}(vt,t))]\,\delta(x - vt), \quad (13)$$

where k and c are spring and dashpot coefficients and $z(t)$ is the vertical displacement of the mass. In this case, we have an additional degree of freedom and, thus, need an additional equation of motion,

$$m\ddot{z} = -k(z(t) - w(vt,t)) - c(\dot{z}(t) - \dot{w}(vt,t)). \quad (14)$$

Note that, $z(t) = 0$ corresponds to the position of the static equilibrium of the spring-mass system as the spring is loaded by the vehicle weight.

The moving oscillator model takes into account the dynamics of the interaction of the bridge with the moving vehicle and, thus, more accurately models a real problem. In this case, we also face the interesting problem of what happens when the oscillator frequency matches the fundamental frequency of the bridge.

4. Finally, the *moving MDOF model* accounts for the facts that (i) the vehicle is not a lumped mass but

rather consists of several elastically interacting components and (ii) it interacts with the structure carrying it at several contact points. Formally, the equation for the contact force can be written here much the same as for the SDOF oscillator (13), assuming that the force f_c is an m-vector, where m is the number of contact points, k and c are stiffness and damping matrices of the vehicle interaction with the ground, P is the vector of weight distribution over the contact points, z is an m vector of "contact" coordinates, and the delta-function is replaced by an m-vector of delta-functions of appropriate arguments. Equation (14) is replaced by a system of n second-order differential equations governing vehicle vibration, where n is the number of degrees of freedom of the vehicle model. The solution procedure for both models is the same, the only difference being that the number of differential equations is greater.

Thus, there is little difference between the moving oscillator and moving MDOF system problems from the standpoint of their formulation and solution. The advantage of the MDOF formulation is that it allows one to more accurately model a real vehicle. Its basic disadvantage is that the results obtained are difficult to analyze because of the large number of the parameters involved.

3.1 Comparison of Vehicle Models for the Case of a Smooth Road Surface and Zero Initial Conditions

As already noted, in bridge related applications the solutions to the moving force and moving mass problems are almost identical. It turns out that, if we consider a smooth bridge profile without any road surface irregularities and assume zero initial conditions for the bridge and vehicle, the solutions to the moving oscillator and moving MDOF system problems are very close to the former solutions as well, even for the case when the vehicle eigenfrequencies are close to those of the bridge. This conclusion is substantiated by results of our numerical experiments as well as by results reported in the literature and is explained as follows: If the initial conditions are zero and the bridge surface is smooth, no sizeable vehicle vibrations are excited due to the finiteness of the passage time and relatively small amplitude of the bridge vibration (compared to the vehicle vibration excited by typical road surface irregularities). Then, the dynamic components of the contact forces are small compared to the vehicle weight, and the bridge vibration is determined mainly by the vehicle weight.

Two next two examples illustrate this. For the vehicle, we used the quarter-car model considered in [9] with the following mass distribution: $m_1 = 3.6\times10^4$ kg (car body) and $m_2 = 4.0\times10^3$ kg (axle group). The suspension and tire stiffness coefficients were taken less than those in [9] to reduce the eigenfrequencies of the vehicle (i.e., to make them closer to those seen in practice): $k_1 = 8.0\times10^6$, and $k_2 = 2.4\times10^7$ N/m. The body-bounce and axle-hop frequencies of this model are 2.05 Hz and 14.3 Hz, respectively. We intentionally set damping in the vehicle model equal to zero, since the difference between the moving oscillator and moving force (mass) models is the most pronounced in the undamped case.

The bridge was modelled as a simply supported, uniform, proportionally damped Euler–Bernoulli beam with mass per unit length $\rho = 1.2\times10^4$ kg/m and bending stiffness $EI = 1.275\times10^{11}$ Nm^2 [7, 9]. We considered two beams the lengths of which, 19 m and 50 m, respectively, were chosen to make their fundamental frequencies match the vehicle eigenfrequencies. The short-span bridge has a fundamental frequency of 14.2 Hz, which is very close to the vehicle axle-hop frequency. The fundamental frequency of the long-span bridge is 2.05 Hz, which perfectly matches the frequency of the vehicle body bounce. The damping in both bridge models was set equal to approximately 2%, which is typical.

The solutions to the moving 2DOF oscillator traversing the short-span and long-span beams at a speed of 25 m/s are depicted in Figs. 2 and 3, respectively, by the solid line. The corresponding moving force solutions (dotted line) were obtained by replacing the 2DOF model by the moving constant force equal to the vehicle weight. The dashed lines correspond to the moving mass solutions (the 2DOF model is replaced by the moving, rigidly attached, body of mass equal to the mass of the whole vehicle).

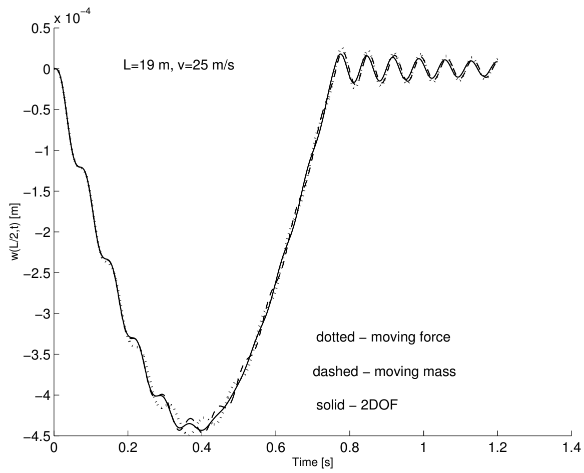

Figure 2: Mid-span deflection of the short-span beam due to different vehicle models traversing the beam at the speed 25 m/s (smooth surface)

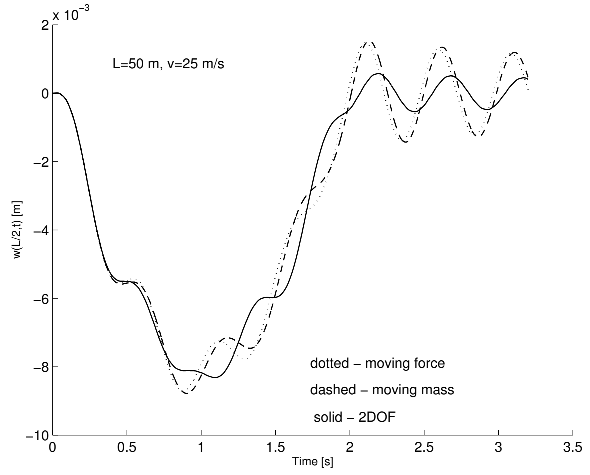

Figure 3: Mid-span deflection of the long-span beam due to different vehicle models traversing the beam at the speed 25 m/s (smooth surface)

As can be seen, the variance among all three solutions is small from a practical standpoint and can be neglected. The dynamic load factor (DLF), defined here as the maximum ratio of the dynamic component of the contact force to the vehicle weight, is very small (several per cent) in both cases, which explains why the solutions for the 2DOF model and moving force model are very close. Note that we have considered the "worst" cases, where the vehicle and bridge eigenfrequencies match well. For beams of other lengths, the variation in the three solutions will be even less.

Two basic conclusions can be derived from the above discussion and illustrations.

1. In the case of smooth bridge surface and zero initial conditions, we can use the the simplest moving force model to get an adequate approximation to the moving vehicle problem. As noted in the DIVINE report [10], "for a smooth profile, the influence of the truck suspension is insignificant."

2. The case of smooth bridge surfaces is of little interest, which implies that we must account for road surface irregularities and consider nonzero initial conditions. Indeed, the values of both the DLF and the DI, which is defined as [11]

$$DI = \frac{(\delta_{dyn} - \delta_{static})}{\delta_{static}} \times 100\%$$

(where δ_{dyn} and δ_{static} are peak dynamic and static deflections, respectively), are around several per cent in this case. On the other hand, results of field or numerical experiments presented in some publications (e.g., [10–12]) report considerably greater values of the DI (more than 100%). These large values cannot be obtained in the framework of an MDOF vehicle model traversing a bridge with a smooth surface.

3.2 The Case of Rough Bridge Surface

We see from the above discussions that high-magnitude bridge vibration cannot result from a smooth bridge surface and zero initial conditions. The case of non-zero bridge initial conditions has been examined in [13]. It was shown there that the magnitude of the force acting on the bridge from the oscillator depends linearly on the vehicle velocity and eigenfrequency. Given low vehicle velocities, considerable interaction forces may arise only in the case of a high vehicle eigenfrequency. In this case, though, a high-magnitude force cannot cause high-magnitude bridge vibration [13] since the fundamental bridge frequency is generally much lower. Thus, although nonzero beam initial conditions may result in considerable dynamic effects in certain circumstances, it is unlikely that they, by themselves, can cause high-magnitude bridge response.

Note that *the only source of non-zero vehicle initial conditions is road irregularities on the approaches to the bridge.* Thus, we may conclude that *high values of the DI measured in some field experiments can be explained only by the presence of road irregularities on the bridge and its approaches.* Then it follows that the examination of the effect of road irregularities is crucial to the analysis of high-magnitude bridge vibration.

Consider the applicability of the moving vehicle models discussed above to the case of a rough or uneven bridge surface. It is evident that the moving force model is not applicable at all since, by definition, the force acting on the bridge is constant (i.e., does not depend on the road profile). The moving mass model is not appropriate either. Indeed, it is an idealization of the moving oscillator model obtained by assuming infinitely large stiffness of the coupling between the vehicle and road and is not appropriate in problems where the interaction with the road plays an important role. Although the force acting on the bridge from the mass depends on the road profile, it has nothing to do with the contact forces arising in real vehicles. The former is the inertia force and depends only on the mass velocity (for a given mass and road profile), whereas real contact forces describe vehicle–road interaction and depend on the vehicle suspension and tire characteristics. Moreover, as shown in [13], the moving mass model is physically incorrect if the function describing the road profile is not smooth (i.e., the slope has jumps), in particular when the initial conditions of the bridge with simple supports on its ends are nonzero.

Then, it follows that *only oscillator models (SDOF os-*

cillator or MDOF system) can be used for modelling a vehicle traversing the bridge with nonsmooth road surface. Generally, the MDOF model is more appropriate: irregularities of different sizes excite vehicle vibrations at different frequencies, and the MDOF model is capable of capturing this phenomenon. However, in many applications, the use of an SDOF vehicle model is still justified and results in almost no loss of accuracy. This is especially true if we are interested in high-magnitude bridge vibration. The point is that bridge vibration with high magnitude can arise only when there is a high-magnitude component in the contact force with its frequency matching the fundamental frequency of the bridge. In this case, other components of the contact force associated with vehicle vibrations at different frequencies have little effect on the bridge and can be neglected (even if they are not small). Hence, we can replace the MDOF model by an SDOF oscillator with the appropriate eigenfrequency.

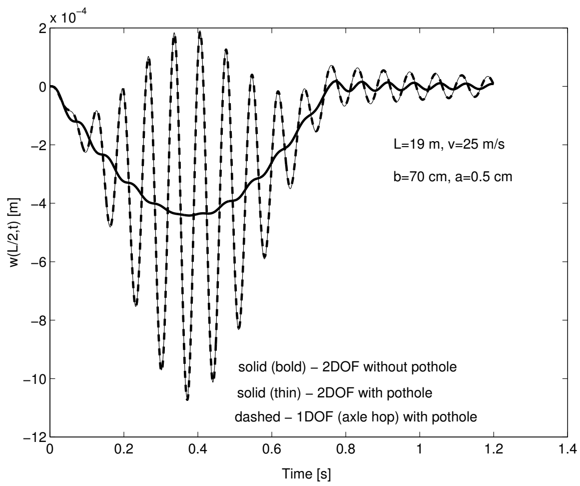

Figure 4: Mid-span deflection of the short-span beam due to a vehicle moving at the speed 25 m/s and entering the beam after passing a short-wavelength pothole obtained with the use of the 2DOF model and 1DOF axle-hop oscillator.

The two following figures illustrate this. We used the same—short-span and long-span—beam models for the bridge and the same 2DOF vehicle model as in the previous illustrations. The bridge profiles were assumed smooth, but we placed a pothole (surface irregularity with the shape described by the cosine function) on the bridge approach immediately before its left end to excite vehicle vibration.

It can be shown that the magnitude of the contact force arising from the passage of a pothole by an SDOF oscillator depends on the oscillator eigenfrequency, velocity and the pothole width, is described by a unique function of one variable, and depends linearly on the pothole depth. This function can be used to evaluate Fourier coefficients of the contact forces arising in the MDOF case. For a given vehicle and velocity, shorter potholes excite mainly axle-hop vibration, whereas longer potholes result in large body bounce. For the 2DOF models used in our experiments and a velocity of $v = 25$ m/s, potholes of width 0.5–2 m excite mainly axle-hop vibration; for potholes of width 5–8 m, axle hop is negligibly small, but the body-bounce is close to the maximum.

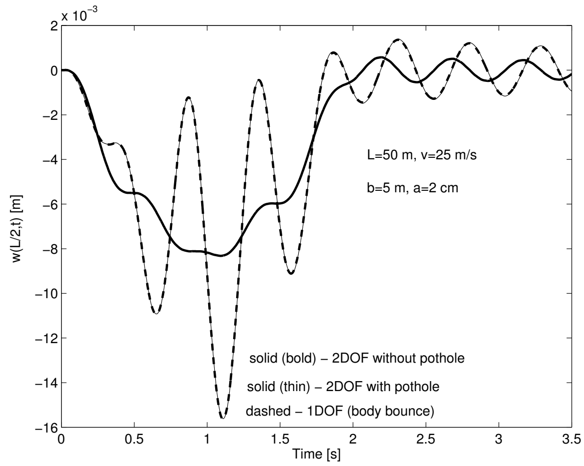

Figure 5: Mid-span deflection of the long-span beam due to a vehicle moving at the speed 25 m/s and entering the beam after passing a long-wavelength pothole obtained with the use of the 2DOF model and 1DOF body-bounce oscillator.

High-magnitude vibrations of the short-span and long-span beams considered can take place only if the frequency of the contact force is around 2 and 14 Hz, respectively, and the magnitude of this force is large enough. To model this, we placed a pothole 70 cm wide and 0.5 cm deep on the short-span beam and a pothole 5 m wide and 2 cm deep on the long-span beam. The solutions obtained by means of the 2DOF model are depicted by the thin solid lines. For comparison, the bold lines show the solutions corresponding to the case where there are no potholes. As can be seen, in both cases, the magnitude of the bridge vibrations is considerably higher than in the "smooth" case, with DLFs being equal to 27% and 43% (DIs more than 100% in both cases), respectively.

To check the quality of the approximation by a reduced model, the 2DOF model was decomposed into two independent—axle-hop and body-bounce—oscillators, and each problem was re-examined. In the case of the short-span bridge (Fig. 4), the 2DOF oscillator was replaced by the axle-hop oscillator, and, for the long-span bridge (Fig. 5) by the body-bounce oscillator. The corresponding solutions are depicted in the figures by the dashed lines. As can be seen, they perfectly coincide with the solutions obtained with the use of the full model.

Thus, when examining high-magnitude bridge vibration, it often suffices to consider SDOF oscillator vehicle models, which not only reduces computational effort, but also facilitates the analysis of the results. Replacement of an MDOF model by an SDOF oscillator is not a trivial exercise, especially in the case of a non-proportionally damped vehicle. The technique for this decomposition will be fully discussed in a future paper [14].

4. CONCLUSION

We have examined the problem of a moving load or subsystem traversing a continuum by, first, developing a comprehensive mathematical statement of the problem and then providing a general method for its solution. The moving loads were classified to the nature of their interactions with the continuum, and the applicability and limitations of each were discussed for both smooth and rough surfaces on the continuum. In the context of the highway bridge problem, we concluded from our studies that high magnitude bridge vibration cannot result from the case of a smooth bridge surface and zero initial conditions, even when a natural frequency of the vehicle coincides with a mode of the bridge. The high-magnitude bridge responses observed in field tests can be explained by irregularities such as potholes on the bridge or its approaches, and only oscillator models, SDOF or MDOF depending upon the particular circumstances, can be used to effectively model these problems.

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