

## CE243A- Behavior and Design of RC Elements

### Assignment #1: Moment-curvature response for unconfined concrete

Due date: 11 October 2004.

1.1 Prepare a spreadsheet to compute the moment versus curvature response for beams (a) and (b) given below, both by hand calculations (for cracking, yielding, and for  $\epsilon_c = 0.003$  with a Whitney stress block) and using a spreadsheet.

- For concrete in compression, use the Hognestad stress-strain relation with  $\epsilon_0 = 0.002$  and  $f'_c = 4$  ksi (0.002, 4), and a linear descending branch defined by (0.0035,  $0.85f'_c$ ). Neglect the contribution of concrete in tension. Use equations to define each region of the stress strain curve (3 regions: (i) prior to peak stress, (ii) linear descending branch, and (iii) zero stress).

$$0 \leq \epsilon_c \leq \epsilon_0 \quad \mathbf{f}_c = f'_c \left[ \frac{2\epsilon_c}{\epsilon_0} - \left( \frac{\epsilon_c}{\epsilon_0} \right)^2 \right]$$

$$\epsilon_0 \leq \epsilon_c \quad \mathbf{f}_c = f'_c \left( 1 - 0.15 \frac{\epsilon_c - \epsilon_0}{0.0035 - \epsilon_0} \right) \geq 0$$

- For the reinforcing bars, consider the following relations:

$$0 \leq \epsilon_s \leq \epsilon_y \quad \mathbf{f}_s = E_s \epsilon_s$$

$$\epsilon_y \leq \epsilon_s \leq \epsilon_{sh} \quad \mathbf{f}_s = f_y$$

$$\epsilon_{sh} \leq \epsilon_s \leq \epsilon_f \quad \mathbf{f}_s = f_y + (\epsilon_s - \epsilon_{sh}) E_{sh}$$

- Verify that your equilibrium condition is correct for beam (a) for an extreme fiber compressive strain of 0.003. That is, check equilibrium for the spreadsheet “by hand” and document your results.
- Compare your results with results obtained for a “hand solution” for points at cracking, yielding, and ultimate (for a Whitney stress block).

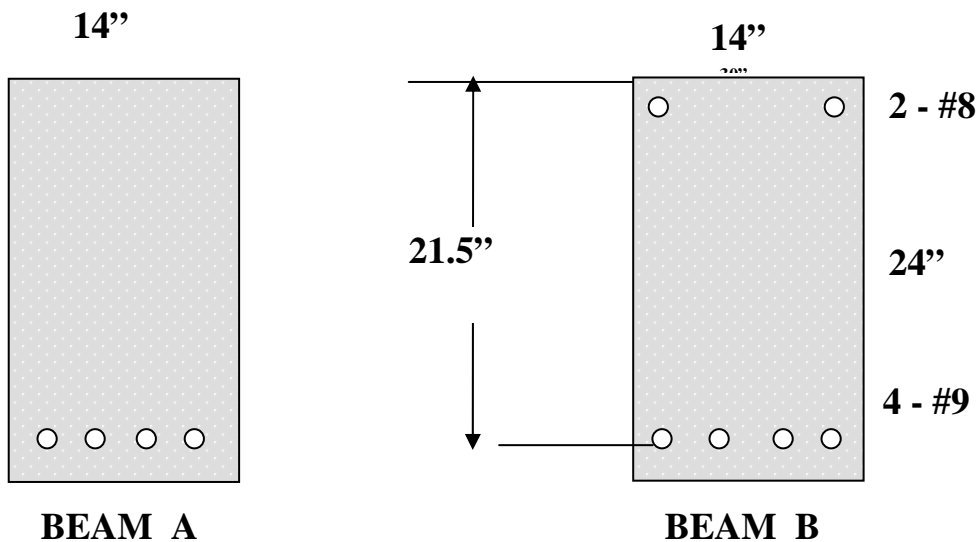
Your spreadsheet should be fairly simple. I would suggest using a numerical integration scheme (e.g., Midpoint Rule, Trapezoidal Rule, Simpson’s Rule) to compute the magnitude and location of the resultant compression force. A potential solution strategy would be to:

- Specify an extreme fiber compressive strain (e.g., 0.0005, 0.001, 0.0015, 0.002, 0.003, 0.005, 0.01, that is, monotonically increasing)
- Assume a neutral axis depth (e.g., you might start with 20% of the section depth)
- Partition the compression region (defined once the neutral axis is assumed) into an arbitrary number of slices (e.g., 20 equal depth slices; Neutral axis depth = 5 inches, with 20 slices, each slice is 0.25 inches deep).
- Given the extreme fiber compressive strain and the assumed neutral axis depth, draw the strain gradient for the section. Determine the strain at the endpoints (or midpoint,

depending on how you decide to iterate) to perform the numerical integration and find the resultant compressive force on each 0.25 inch slice. With enough slices, you can assume the resultant for each slice is at the middle of the slice, or you could be a bit more elaborate and find the centroid of a trapezoid for an assumed linear distribution for each slice.

- (5) Sum the compressive force for each slice and the sum of the compressive force times a distance from a reference point to find the resultant compressive force as well as the location of the resultant.
- (6) Using the strain gradient, compute the strain, stress, and the resulting tension force in the reinforcing steel layers (e.g., tension steel, compression steel)
- (7) Check if  $C=T$ , and iterate until you achieve equilibrium (return to step (2) and modify the neutral axis depth based on the unbalance in C and T).
- (8) Once equilibrium is achieved, compute the moment capacity and the curvature for the equilibrium condition.
- (9) Continue for a sufficient number of points to draw the moment versus curvature diagram for strains between 0.00025 and 0.01.

It is acceptable to conduct the iteration “manually”, that is, for a given extreme fiber compressive strain, input a neutral axis depth and have your spreadsheet check equilibrium, and “manually” input a new guess for neutral axis depth based on the equilibrium check until you reach equilibrium. (That is, it is not necessary to construct a spreadsheet that will automate the iteration process).



Note: The beam is 14" x 24", and the dimension from top of beam to centerline of the tension steel (4 - #9) is 21.5". The distance from the top of the beam to the centerline of the compression steel (2-#8) is 2.5 inches.